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# Experimental and Simulation Predicted Crack Paths For AL-2024-T351 Under Mixed-Mode I/II Fatigue Loading Using An Arcan Fixture

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EXPERIMENTAL AND SIMULATION PREDICTED CRACK PATHS FOR AL-2024-T351 UNDER MIXED-MODE I/II FATIGUE LOADING USING AN ARCAN FIXTURE

by

Eileen Miller

Bachelor of Science  
University of South Carolina, 2011

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Submitted in Partial Fulfillment of the Requirements

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2013

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## ABSTRACT

Mixed mode I/II fatigue experiments and simulations are performed for an Arcan fixture and a 6.35mm thick Al-2024-T351 specimen. Experiments were performed for Arcan loading angles that gave rise to a range of Mode I/II crack tip conditions from  $0 \leq \Delta K_{II}/\Delta K_I \leq \infty$ . Measurements include the crack paths, loading cycles and maximum and minimum loads for each loading angle. Simulations were performed using three-dimensional finite element analysis (3D-FEA) with 10-noded tetrahedral elements via the custom in-house FEA code, CRACK3D. While modeling the entire fixture-specimen geometry, a modified version of the virtual crack closure technique (VCCT) with automatic crack tip re-meshing and a maximum normal stress criterion was used to predict the direction of crack growth. Results indicate excellent agreement between experiments and simulations for the measured crack paths during the first several millimeters of crack extension.

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## LIST OF SYMBOLS

$\sigma_{\theta\theta max}$  Maximum circumferential stress defined in polar coordinates around the crack tip.

$K$  Stress intensity factor.

$K_I$  Stress intensity factor for Mode I.

$K_{II}$  Stress intensity factor for Mode II.

$K_{III}$  Stress intensity factor for Mode III.

$\sigma$  Far field tensile stress.

$a$  Crack length.

$w$  Specimen width.

$\Delta K$  Difference in maximum applied stress intensity factor and minimum applied stress intensity factor for fatigue loading.

$K_{max}$  Maximum applied stress intensity factor for fatigue loading.

$K_{min}$  Minimum applied stress intensity factor for fatigue loading.

$R$  Loading ratio, also known as  $R$ -ratio.

$\sigma_{max}$  Maximum applied far-field tensile stress for fatigue loading.

$\sigma_{min}$  Minimum applied far-field tensile stress for fatigue loading.

$N$  Number of loading cycles.

$da/dN$  Crack growth rate which is the differential amount of crack growth per number of stress cycles.

$C$  Material and  $R$ -ratio dependent proportionality constant for Paris' Law.

$m$  Material and  $R$ -ratio dependent power constant for Paris' Law.

- $\Delta K_{TH}$  Threshold  $\Delta K$ . The value below which the fatigue crack will not propagate.
- $K_c$  Fracture toughness. The value which, if  $K_{max}$  exceeds, rapid crack propagation ensues and final fracture occurs.
- $F_I$  Function of loading, geometry, and crack orientation for determining  $K_I$  empirically.
- $F_{II}$  Function of loading, geometry, and crack orientation for determining  $K_{II}$  empirically.
- $t$  Specimen thickness.
- $P$  Pin load applied using Arcan fixture.
- $\alpha$  Angle from the vertical direction to the first segment of a kinked crack.
- $\beta$  Angle from the first segment of a kinked crack to the second segment.
- $b$  Crack length for the second segment of a kinked crack.
- $\Phi$  Loading angle for Arcan fixture.
- $h$  Distance from crack to uniform tensile stress for Tada's empirical solution for  $K_I$ .
- $\Delta K_{eq}$  Equivalent  $\Delta K$  for the mixed mode loading condition.
- $\gamma, \gamma_1, \gamma_2$  Parameters for  $\Delta K_{eq}$ .
- $x_{i,f}^{\Phi}$  The  $i^{\text{th}}$  x-position in pixels of the crack path for the front of the specimen for loading case  $\Phi$ .
- $x_{i,b}^{\Phi}$  The  $i^{\text{th}}$  x-position in pixels of the crack path for the back of the specimen for loading case  $\Phi$ .
- $y_{i,f}^{\Phi}$  The  $i^{\text{th}}$  y-position in pixels of the crack path for the front of the specimen for loading case  $\Phi$ .
- $y_{i,b}^{\Phi}$  The  $i^{\text{th}}$  y-position in pixels of the crack path for the back of the specimen for loading case  $\Phi$ .
- $\Delta x_i^{\Phi}$  The standard deviation in pixels for all the x-positions of the crack path for the front of the specimen for loading case  $\Phi$ .

- $\Delta y_i^\Phi$  The standard deviation in pixels for all the y-positions of the crack path for the front of the specimen for loading case  $\Phi$ .
- $s_{v,f}^\Phi$  The average vertical scale factor used for converting crack path y-position from pixels to meters for the front of the specimen for loading case  $\Phi$ .
- $s_{h,f}^\Phi$  The average horizontal scale factor used for converting crack path x-position from pixels to meters for the front of the specimen for loading case  $\Phi$ .
- $s_{v,b}^\Phi$  The average vertical scale factor used for converting crack path y-position from pixels to meters for the back of the specimen for loading case  $\Phi$ .
- $s_{h,b}^\Phi$  The average horizontal scale factor used for converting crack path x-position from pixels to meters for the back of the specimen for loading case  $\Phi$ .
- $\Delta s_{v,f}^\Phi$  The standard deviation of the vertical scale factor used for converting crack path y-position for the front of the specimen for loading case  $\Phi$ .
- $\Delta s_{h,f}^\Phi$  The standard deviation of the horizontal scale factor used for converting crack path x-position for the front of the specimen for loading case  $\Phi$ .
- $\Delta s_{v,b}^\Phi$  The standard deviation of the vertical scale factor used for converting crack path y-position for the back of the specimen for loading case  $\Phi$ .
- $\Delta s_{h,b}^\Phi$  The standard deviation of the horizontal scale factor used for converting crack path x-position for the back of the specimen for loading case  $\Phi$ .
- $X_{i,f}^\Phi$  The average  $i^{\text{th}}$  x-position in meters of the crack path for the front of the specimen for loading case  $\Phi$ .
- $X_{i,b}^\Phi$  The average  $i^{\text{th}}$  x-position in meters of the crack path for the back of the specimen for loading case  $\Phi$ .
- $Y_{i,f}^\Phi$  The average  $i^{\text{th}}$  y-position in meters of the crack path for the front of the specimen for loading case  $\Phi$ .
- $Y_{i,b}^\Phi$  The average  $i^{\text{th}}$  y-position in meters of the crack path for the back of the specimen for loading case  $\Phi$ .
- $\Delta X_{i,f}^\Phi$  The standard deviation for the  $i^{\text{th}}$  x-position in meters of the crack path for the front of the specimen for loading case  $\Phi$ .
- $\Delta X_{i,b}^\Phi$  The standard deviation for the  $i^{\text{th}}$  x-position in meters of the crack path for the back of the specimen for loading case  $\Phi$ .

- $\Delta Y_{i,f}^{\Phi}$  The standard deviation for the  $i^{\text{th}}$  y-position in meters of the crack path for the front of the specimen for loading case  $\Phi$ .
- $\Delta Y_{i,b}^{\Phi}$  The standard deviation for the  $i^{\text{th}}$  y-position in meters of the crack path for the back of the specimen for loading case  $\Phi$ .
- $\Delta a/\Delta N$  Crack growth rate which is the discrete amount of crack growth per number of stress cycles.
- $K_x$  Spring constant assumed in VCCT used in calculating the force in the x-direction.
- $K_y$  Spring constant assumed in VCCT used in calculating the force in the y-direction.
- $K_z$  Spring constant assumed in VCCT used in calculating the force in the z-direction.
- $F_x$  Reaction force in the x-direction on each node on the crack front and the mid-nodes attached to the element just ahead of the crack front in VCCT.
- $F_y$  Reaction force in the y-direction on each node on the crack front and the mid-nodes attached to the element just ahead of the crack front in VCCT.
- $F_z$  Reaction force in the z-direction on each node on the crack front and the mid-nodes attached to the element just ahead of the crack front in VCCT.
- $u_x$  Nodal displacement in the x-direction.
- $u_y$  Nodal displacement in the y-direction.
- $u_z$  Nodal displacement in the z-direction.
- $G_I$  Strain energy release rate for Mode I.
- $G_{II}$  Strain energy release rate for Mode II.
- $G_{III}$  Strain energy release rate for Mode III.
- $E$  Young's modulus of elasticity.
- $\nu$  Poisson's Ratio.
- $\Delta a$  Crack growth increment.
- $\theta_c$  Angle which the crack will propagate according to MCS criterion.
- $\sigma_{\theta\theta}$  Circumferential normal stress defined in polar coordinates around the crack tip.

$\sigma_{r\theta}$  Shear stress defined in polar coordinates around the crack tip.



## LIST OF ABBREVIATIONS

3D-FEA.....	3 – Dimensional Finite Element Analysis
SIF.....	Stress Intensity Factor
2D-DIC .....	2 – Dimensional Digital Image Correlation
VCCT.....	Virtual Crack Closure Technique
MCS .....	Maximum Circumferential Stress
LT.....	Longitudinal-Transverse
MTS .....	Material Test System
NASA.....	National Aeronautics and Space Administration
CCD .....	Charge-Coupled Device

# CHAPTER 1

## INTRODUCTION

### 1.1 MOTIVATION

The aerospace industry has experience with a range of structural failures, oftentimes due to fatigue cracks in aircraft fuselage components that are exposed to relatively high stress levels during cyclic loading effects incurred during repeated take-off and landing events that lead to fatigue crack initiation at material defects and near stress concentrations. One of the first incidents that raised public awareness occurred in April of 1988 when 18 feet of the fuselage ripped off one of Aloha Airlines' Boeing 737s in midflight. The cabin quickly decompressed resulting in one fatality and eight serious injuries. The National Transportation Safety Board reported that undetected dis-bonding and widespread fatigue damage between rivets led to the failure of a lap joint [1]. A few months later, Continental Airlines found several 30 inch long cracks in a Boeing 737 aircraft in the same general area where damage occurred in the Aloha Airlines incident [2].

Ten years later, in October 1998, structural fatigue cracks in the fuselage of a Boeing 737s were reported [3], prompting the Federal Aviation Administration to propose the Airworthiness Directive [3]. The directive required aircraft with less than 60,000 flight cycles to be inspected initially and then inspected again every 3,000 cycles. Furthermore, aircraft would be required to receive modifications to strengthen the bulkhead before

75,000 cycles. Considering only Boeing 737s in the United States, the estimated cost of the inspections could be up to \$26 million per inspection cycle and \$71 million for modifications [3].

Despite the efforts of airlines and the Federal Aviation Administration to monitor fatigue cracks in aging aircraft, the danger of structural failure in fuselages continues into the 21<sup>st</sup> century. In 2009, fatigue at the top of the fuselage just in front of the vertical tail fin caused a 12 inch hole to rip open midflight, causing decompression of the cabin and an emergency landing of a Southwest Airline Boeing 737 [4]. Southwest Airline had another fuselage failure during flight just two years later. A section near the top of the fuselage, about five feet long and one foot wide, ripped off due to the sudden propagation of fatigue cracks in the skin of the aircraft [5].

The presence of fatigue cracks is not exclusive to commercial jetliners. In 2004, Lockheed Martin made the switch from titanium to aluminum for some structural features of the F-35 [6]. In 2010, fatigue cracks were discovered on the bulkhead of a Lockheed Martin ground test aircraft [6]. Although no structural failure occurred in these cases, the presence of fatigue cracks must be monitored to avoid potential catastrophes.

In fact, fatigue cracks are expected to form in the fuselage of modern airplanes due to repeated (a) pressurization and decompression of the cabin during every flight and (b) loading effects during take-off and landing. Thus, the propagation of cracks into critical joints continues to be an area of concern, especially since such propagation under complex stress states is not completely understood. Although procedures are currently in place to inspect and repair fatigue cracks, the ability to better predict how far a crack will

propagate and in which direction it would grow when subjected to various loading conditions could save millions of dollars in premature inspection and repair, while also identifying the severity of an existing flaw in an aero-structure.

## 1.2 BACKGROUND

As noted in Section 1.1, flaws in aircraft components oftentimes are exposed to complex stress states. For nominally elastic conditions, the crack tip stress states generally are decomposed into three modes of loading which are shown schematically in Figure 1.1. Mode I, the opening mode, is such that the crack surfaces move away from

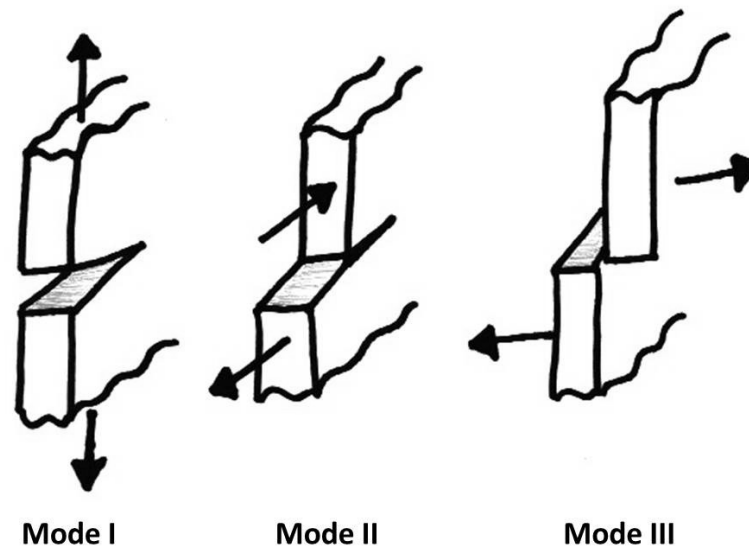


Figure 1.1. The three fracture modes for nominally elastic conditions.

each other and the material directly ahead of the crack is subjected to a dominant tensile stress. Mode II, the in-plane sliding mode, is such that shear loading is applied parallel to the direction of crack growth and the material directly ahead of the crack tip is subjected to a dominant in-plane shear stress. Mode III, the out-of-plane shearing mode, is

designated either out-of-plane or transverse shear, with the crack surfaces moving parallel to and across each other. Mode I crack tip conditions are generally the dominant influence on fatigue crack propagation in most aerospace metallic components (e.g., aluminum, titanium).

Crack propagation under Mode I loading is reasonably well understood [7]. Using a maximum circumferential stress (MCS) criterion, the predicted and actual crack trajectories during fatigue loading are perpendicular to the local  $\sigma_{\theta\theta \max}$  direction where  $\sigma_{\theta\theta \max}$  is the maximum circumferential stress ahead of the crack tip [8]. This direction nominally coincides with the loading direction when local conditions are not influenced by stress concentrations, material defections/inclusions, or other factors.

For high cycle fatigue, it is generally assumed that the far field stress remains linear elastic, while the local stress also remains mostly linear elastic with a small plastic zone around the crack tip (ideally, the plastic zone size would be no more than one tenth of the thickness of the specimen). The stress intensity factor (SIF),  $K$ , is a value which describes the magnitude of the local elastic stress field and is a function of the stress and geometry of the structure and crack. For Mode I loading,  $K_I$  is defined by [7]

$$K_I = \sigma\sqrt{\pi a} f(a/w) \quad [1.1]$$

where  $\sigma$  is the far field tensile stress;  $a$  is the crack length; and  $f(a/w)$  is a parameter which depends on the geometry of the specimen and crack orientation. Since the loading is cyclic for fatigue studies, the loading parameter,  $\Delta K$ , is considered the driving force for fatigue crack propagation and is defined as follows;

$$\Delta K = K_{\max} - K_{\min} \quad [1.2]$$

where *max* and *min* refer to the maximum and minimum applied values of *K*. Another loading parameter that has been shown to be important in fatigue studies is the loading ratio, also known as the R-ratio. The R-ratio is defined as

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{K_{min}}{K_{max}} \quad [1.3]$$

Therefore  $\Delta K$  can be expressed as

$$\Delta K = (1 - R)K_{max} \quad [1.4]$$

The use of  $\Delta K$  as the primary driving force in high cycle fatigue was introduced by Paul Paris in his pioneering work [9]. Paris' Law defines the relationship between  $\Delta K$  and the differential amount of crack extension per stress cycle,  $da/dN$ , and is written;

$$\frac{da}{dN} = C \Delta K^m \quad [1.5]$$

Paris' Law parameters, *C* and *m*, are determined experimentally for each material and may or may not be dependent on the loading ratio, *R*.

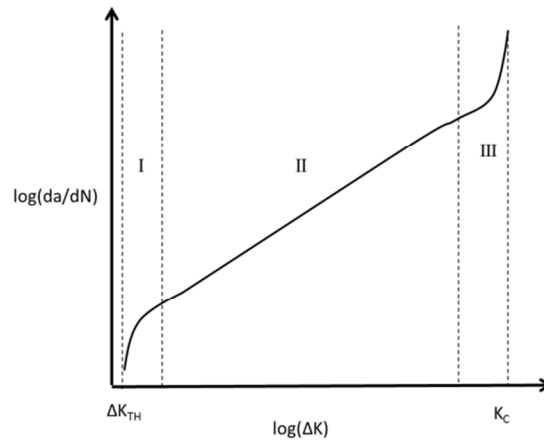


Figure 1.2. A schematic of a typical log-log plot of  $da/dN$  vs.  $\Delta K$ .

As shown in Figure 1.2, a typical plot of the fatigue crack propagation process has three regions. Below threshold,  $\Delta K_{TH}$ , the crack will not propagate. Then in region I, the crack growth begins to transition to region II where crack propagation occurs in a manner that is predicted by Eq 1.5. In region III, the crack growth again transitions as  $K_{max}$  approaches the fracture toughness,  $K_c$ . When  $K_{max} \geq K_c$ , rapid crack propagation ensues until final fracture occurs.

Now consider the case where a crack is under mixed-mode loading, that is, under any combination of two or more loading types (see Figure 1.1). For the combination of Mode I and Mode II loading conditions, methods for obtaining a mixed-mode I/II stress state experimentally when applying uniaxial tensile loading include (a) use of kinked cracks, (b) use of cracks propagating away from a hole, and (c) use of an Arcan fixture [9-18].

Independent mixed mode loading studies by both Zhang *et al.* [10] and Lopez-Crespo *et al.* [11] have used an Arcan fixture to statically load an existing crack while 2D digital image correlation (2D-DIC) was used to determine  $K_I$  and  $K_{II}$  from measured displacement fields around the crack tip for various degrees of Mode I/II loading. Zhang *et al.* used an Arcan fixture with a through thickness edge notch as the one shown in Figure 1.3. Lopez-Crespo *et al.* used the same fixture with a center notched specimen. Experimental SIFs were compared to values obtained from empirical expressions for the Arcan fixture. The edge cracked solution takes the following form [12] .

$$K_I = F_I \frac{P}{wt} \sqrt{\pi a}, K_{II} = F_{II} \frac{P}{wt} \sqrt{\pi a} \quad [1.6]$$

where t is specimen thickness and  $F_I$  and  $F_{II}$  are provided graphically as a function of various loading angles and  $0.45 \leq a/W \leq 0.7$  [12]. There are some limiting factors for this

model. First, it is only valid for a range of relatively large cracks. Secondly, it only considers straight cracks. Finally, it is only valid for static cracks. Thus, this model can only be used for determining the kinking angle for the initial crack propagation event.

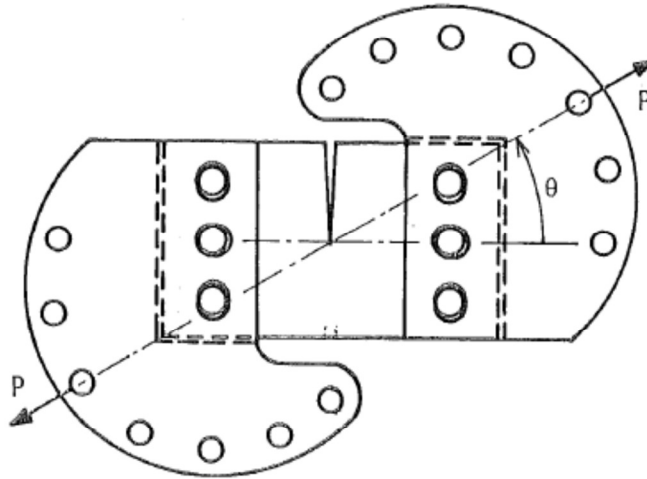


Figure 1.3. Diagram of Arcan fixture and specimen.

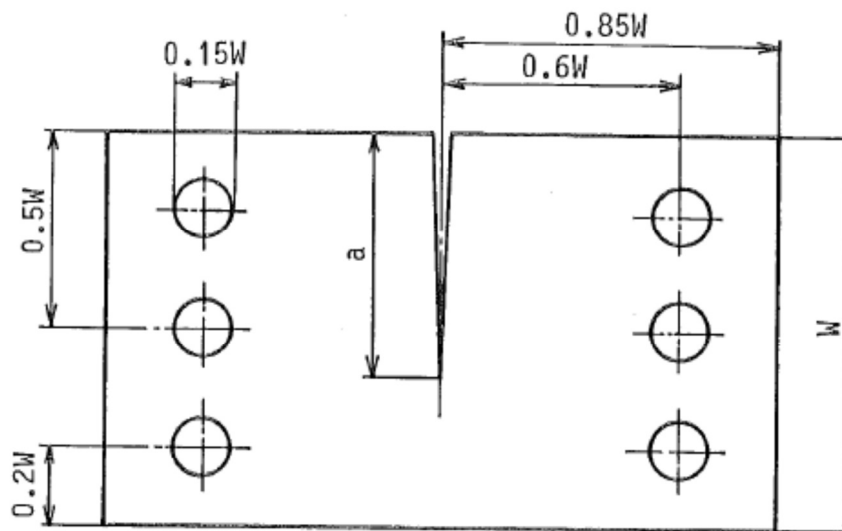


Figure 1.4. Specimen for Arcan fixture with single edge crack.

For the case of a kinked crack under uniform tensile loading (see Figure 1.5), another empirical model exists [12]



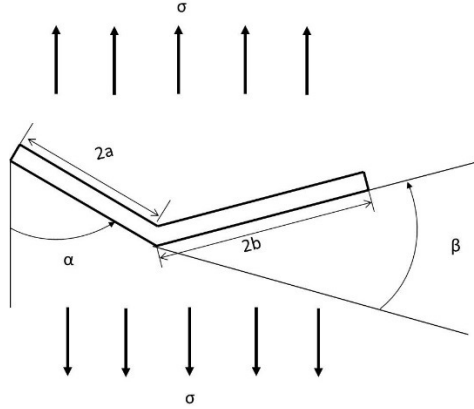


Figure 1.5. Schematic of kinked crack.

$$K_I = \sigma\sqrt{\pi a}F_I(\alpha, \beta, \frac{b}{a}), \quad K_{II} = \sigma\sqrt{\pi a}F_{II}(\alpha, \beta, \frac{b}{a}) \quad [1.7]$$

$$\begin{pmatrix} F_I(\alpha, \beta, \frac{b}{a}) \\ F_{II}(\alpha, \beta, \frac{b}{a}) \end{pmatrix} = \frac{1 - \cos 2\alpha}{2} \begin{pmatrix} F_I^1(\beta, \frac{b}{a}) \\ F_{II}^1(\beta, \frac{b}{a}) \end{pmatrix} - \cos 2\alpha \begin{pmatrix} F_I^2(\beta, \frac{b}{a}) \\ F_{II}^2(\beta, \frac{b}{a}) \end{pmatrix} - \frac{\sin 2\alpha}{2} \begin{pmatrix} F_I^3(\beta, \frac{b}{a}) \\ F_{II}^3(\beta, \frac{b}{a}) \end{pmatrix} \quad [1.8]$$

$$\begin{pmatrix} F_I^k(\beta, \frac{b}{a}) = \sum_{n=0}^2 F_{I,n}^k(\beta) \left(\frac{b}{a}\right)^n \\ F_{II}^k(\beta, \frac{b}{a}) = \sum_{n=0}^2 F_{II,n}^k(\beta) \left(\frac{b}{a}\right)^n \end{pmatrix} \text{ for } k = 1, 3 \quad [1.9]$$

$$\begin{pmatrix} F_I^k(\beta, \frac{b}{a}) = \sum_{n=0}^2 F_{I,n}^k(\beta) \left(\frac{b}{a}\right)^{n+1/2} \\ F_{II}^k(\beta, \frac{b}{a}) = \sum_{n=0}^2 F_{II,n}^k(\beta) \left(\frac{b}{a}\right)^{n+1/2} \end{pmatrix} \text{ for } k = 2 \quad [1.10]$$

where  $F_{I,n}^k$  and  $F_{II,n}^k$  for  $n=0, 1, \text{ and } 2$  and  $k=1, 2, \text{ and } 3$  are provided in a table for  $0^\circ \leq \beta \leq 180^\circ$  [12]. Equation 1.7 is valid for  $0 \leq \frac{b}{a} \leq 0.2$ . Limitations to this model are that (a) it is applicable to kinked cracks in an infinite plate, (b)  $b \ll a$ , and (c) the loading must be distributed in such a way that uniform stress is applied to the region in which the crack is located. [12]

Gaylon et al. [13] performed fatigue tests using the Arcan fixture. In this study, the authors determined the crack growth trajectory for various degrees of mixed-mode I/II loading. Their results indicate that the crack trajectory is curvilinear and the stress distribution applied to the crack is non-uniform. Therefore, the two empirical models

provided by Murakami and discussed previously are not applicable to quantify the mixed mode SIFs. Also, the measured crack trajectories suggest that for all combinations of Mode I/II loading, the fatigue cracks propagate in a manner that was locally dominated by  $K_I$ , while no crack propagation occurred for the pure Mode II loading case. However, there was such large scatter in the experimental data that it is difficult to definitively identify the trends. One cause of the inconsistency in the results was determined to be the three pin loading configuration used by the authors. It was suggested that future studies use only one pin for fixing the Arcan fixture to the test stand [14]; the use of a single pin is consistent with the work of Amstutz, Boone and others at the University of South Carolina [13, 14, 18, 21-23].

Chao et al. [15] also used the Arcan fixture with the one-pin configuration to study fatigue crack propagation under various mixed-mode loading conditions. Crack trajectories were compared to stable tearing results obtained under mixed-mode monotonic loading conditions. It was observed that cracks under fatigue loading propagate in a local Mode I direction for all loading cases including pure Mode II, unlike Gaylon's results. In Chao's studies, the amount of crack growth in fatigue for  $\Phi=75^\circ$  and  $90^\circ$  was quite small, indicating that the crack surfaces interfered after a small amount of crack extension and impeded further crack growth. For stable tearing, after Mode II loading becomes dominant, cracks in aluminum alloys tended to propagate in the local shear direction; that is, approximately parallel to the direction of the pre-crack. This transition from Mode I dominated crack growth to Mode II dominated crack growth under stable tearing conditions is consistent with results obtained by Amstutz et al [16] [17]. In Amstutz's work, the authors used the Arcan fixture to study mixed Mode I/II

stable tearing crack growth. The results show that for most loading cases, where  $K_{II}/K_I < 1$ , the crack propagates under local Mode I conditions. However, as  $K_I$  approaches zero and  $K_{II}/K_I$  reaches a critical value ( $\Phi=75^\circ$  and  $90^\circ$  for Al 2024-T351), the crack begins to grow in Mode II. While this study included crack propagation, stable tearing occurs outside of the linear elastic range, and results suggest that the Mode II component has different effects in the linear elastic range than it does under elastic-plastic conditions.

Boljanovic [18] performed finite element analysis to model the results of Gaylon et al. The crack trajectories were simulated using MSC/NASTRAN [19] in a step-by-step method while applying the maximum circumferential stress (MCS) criterion to predict crack trajectory. Results of Boljanovic's work agree with Gaylon's experimental crack paths. However, the SIFs were not obtained at each step using the local crack tip field data, but were determined analytically after the simulation was performed since the step-by-step method of crack path prediction is quite time consuming. It is unclear if the analytical solution for the SIFs accounted for curvilinear crack paths.

### 1.3 CURRENT WORK

The objective of the current study is to (a) perform experiments and measure the crack path and (b) perform simulations and predict the fatigue crack path in an aerospace aluminum alloy undergoing applied, far-field mixed-mode I/II conditions. The Arcan fixture will be utilized to achieve far-field mixed-mode I/II conditions in 6.35mm thick Al-2024-T351 specimens. Crack paths, cycle count, and maximum and minimum loads will be measured during experiments, with loading ranging from  $0 \leq K_{II}/K_I \leq \infty$ . Simulations will then be performed using 3D-FEA. Crack trajectories will be predicted using virtual crack closure techniques (VCCT) and a MCS criterion. Local re-meshing

will be used to extend the crack. The whole fixture and specimen will be modeled using 10-noded tetrahedral elements. Predicted crack paths will be compared to the results obtained experimentally, and the results will be discussed.

## CHAPTER 2

### EXPERIMENTAL WORK

#### 2.1 FIXTURE AND SPECIMEN PREPARATION

The Arcan fixture shown in Figure 2.1 was used to achieve mixed-mode I/II loading for discrete values of  $K_{II}/K_I$  in the range  $0 \leq K_{II} / K_I \leq \infty$ . The butterfly-shaped specimen shown in Figure 2.1 is machined to have tight contact with the upper and lower grips along all four straight, angled sides. Once tightly fitted into the grips, the specimen is further tightened into place using ten small bolts; five on the top part of the fixture and five on the bottom part. Around the edges of the stainless-steel grips are pairs of holes located every  $15^\circ$ . With loading angle  $\Phi$  defined as shown in Figure 2.1, the  $\Phi = 0^\circ$  pin holes correspond to nominally Mode I crack conditions and the  $\Phi = 90^\circ$  pin holes represent nominally Mode II crack loading conditions. The fixture was machined from 15-5PH stainless steel with Young's modulus  $= 2.07 \times 10^{11}$  Pa and Poisson's ratio  $= 0.30$ .

As shown in Figure 2.2, each butterfly-shaped specimen is 224.28 mm tall, 275.30 mm wide at the top and bottom of the specimen and 6.35mm thick. Each specimen is manufactured from Al-2024-T351 to form an LT orientation crack configuration (crack is along the transverse direction (T) and perpendicular to the rolling direction (L) in the aluminum specimen) [20] with Young's modulus  $= 7.11 \times 10^{10}$  Pa and Poisson's ratio  $= 0.33$ . A jeweler's saw blade, size 0/6, was used to create an initial through-thickness edge notch 6.35mm long in the width direction on the left side of the specimen in the vertical

center (see Figure 2.2). The front and back surfaces of the specimens were sanded with 600 grit sand paper before final sanding with 800 grit sandpaper to remove small surface defects. Metal polish was used to create a mirror finish on the surfaces for visually tracking crack tip progression during the experiment.

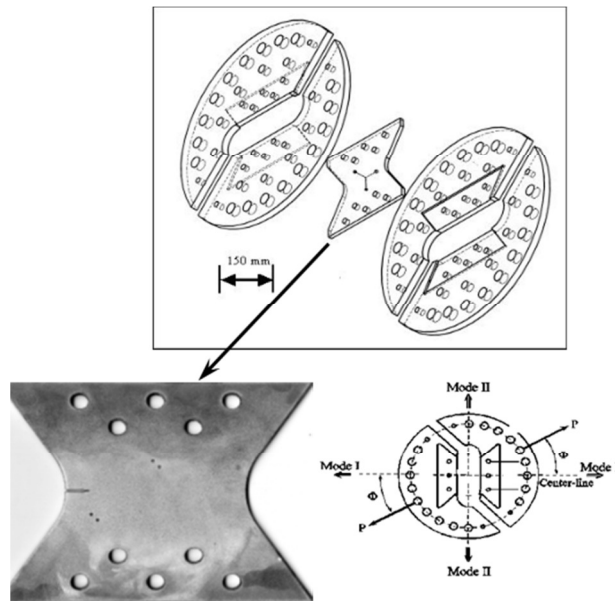


Figure 2.1. Mixed mode I/II Arcan test fixture and butterfly shaped test specimen. Angle  $\Phi=0^{\circ}$  corresponds to far-field tension and  $\Phi=90^{\circ}$  is far-field shear.

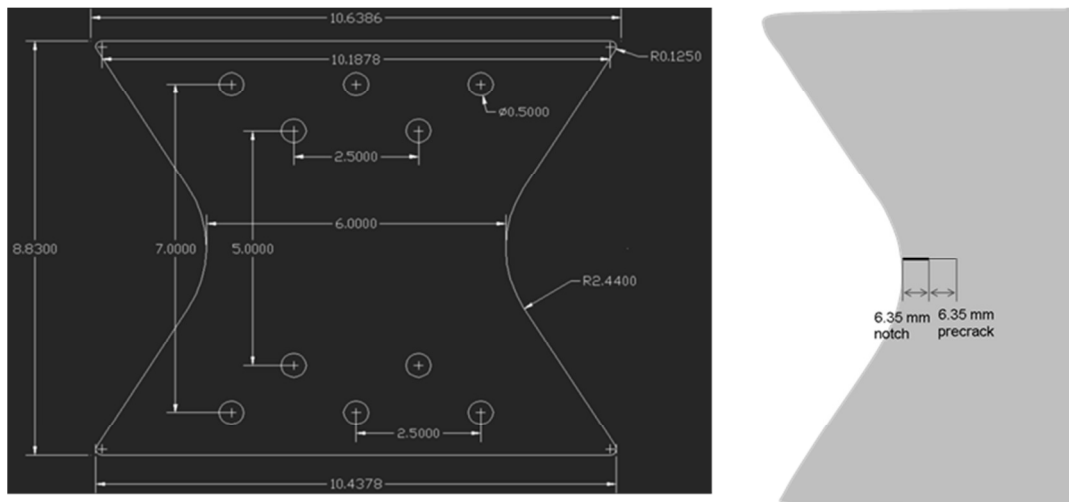


Figure 2.2. (a) Dimensions of specimens in inches (b) Diagram of notch and pre-crack

## 2.2 SETUP

As shown in Figure 2.3, a 50 kip (227 kN) servo-hydraulic Material Test System (MTS) controlled by TestStar II software was used to apply tensile loads in displacement control to the Arcan fixture and specimen. Stainless steel clevises were placed in the hydraulic grips of the MTS test frame, and the fixture was attached with one pin on the top and another pin on the bottom. . The top image of Figure 2.3 is of the complete test set-up with (a) the specimen and Arcan fixture pinned into the clevises of the test stand and (b) microscope objectives and slide apparatus for optical tracking of the propagating crack tip clamped to the test stand. The bottom image of Figure 2.3 shows the set up without the microscope objectives. The backing plate (not visible in Fig 2.3) is attached to the top and bottom pieces of the Arcan fixture and is oriented at 45°.

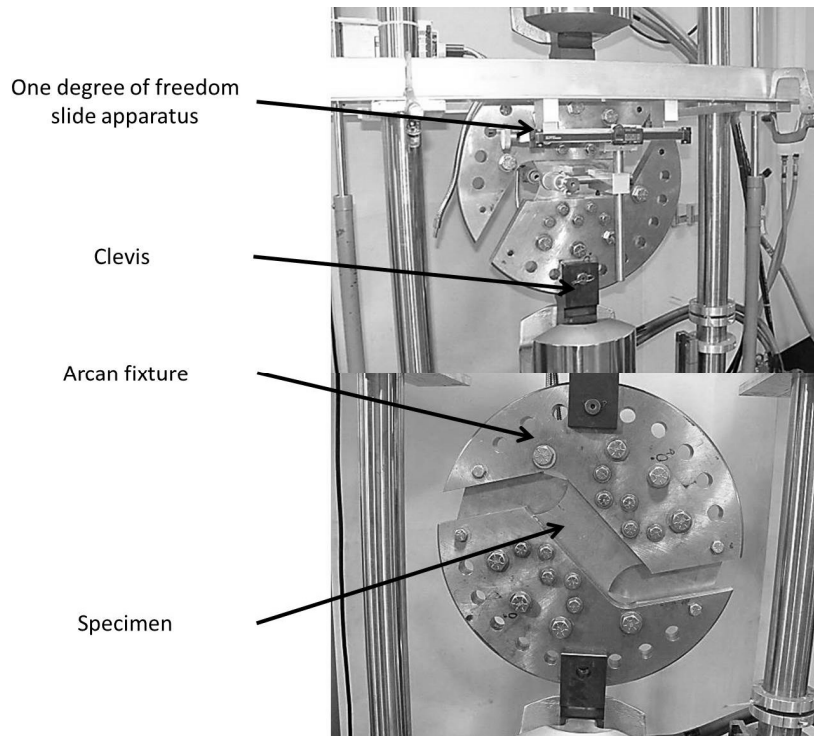


Figure 2.3. Images of experimental set up for a 45° loading angle.

During testing, the crack tip was tracked using the microscope objective and the slide apparatus. The objective is attached to the dual slide apparatus shown in Figure 2.4. The dual slide apparatus was designed and constructed by Mr. Haywood Watts. The apparatus consists of (a) a single, horizontally mounted manual screw driven slide manufactured by Velmex with a digital caliper to provide a metric positional measurement, (b) a second vertically-oriented Velmex slide with digital caliper that was mounted to the horizontal slide. The microscope objective was then connected to the vertical slide. Both vertical and horizontal slides operate independently, allowing for horizontal and vertical measurements of the crack tip position during the fatigue process.

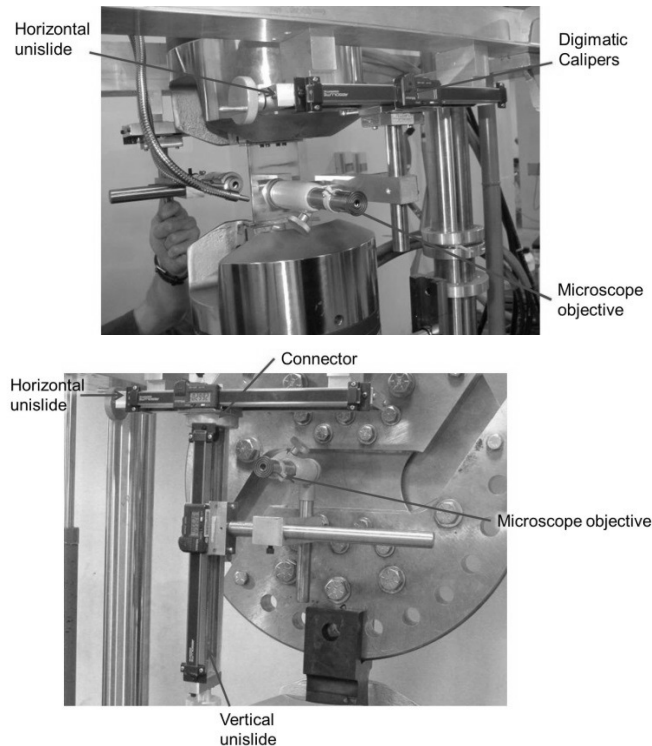


Figure 2.4. (Top) One degree-of-freedom slide apparatus; (Bottom) Two degree-of -freedom slide apparatus.



### 2.3 EXPERIMENTAL PROCEDURE

To mount the notched specimen into the Arcan fixture, it was first bolted into the top and bottom Arcan fixtures that were held in place by a backing plate designed to connect the top and bottom halves of the fixture and keep the assembly from moving during installation of the specimen into the fixture, minimizing initial distortions/stresses in the specimen prior to the experiment. The fixture-specimen-backing plate combination was pinned to the upper clevis, rotated about the pin to align with the bottom clevis and then the lower pin was put in place to fully install the specimen-fixture combination in the MTS test stand. Initially, the specimen is oriented to be in the Mode I configuration. Once fully installed, the backing plate is removed. Then, two sets of dual slide apparatuses were clamped to the test stand – one for tracking the crack on the front of the specimen and the other for tracking the crack on the back of the specimen. The calipers attached to the slides were zeroed at the center of the notch on the edge of the specimen, see Figure 2.5. After everything is in place, the specimens were fatigue pre-cracked an additional 6.35mm for a total crack length of 12.7mm. Fatigue loading was applied in force control according to the loads outlined in the following section at 10Hz.

After pre-cracking, the crack front was marked by cycling at a higher loading ratio ( $R=0.8$  or  $R=0.9$ ), and at about 90% of the pre-crack load. Then the backing plate was reattached to the fixture, the fixture was rotated in the test stand to the appropriate loading angle (e.g. see Figure 2.5). Once the specimen-fixture combination is correctly positioned for the specific loading angle,  $\Phi$ , of interest, the backing plate was again removed and the microscope objectives were repositioned. The microscopes were re-

zeroed at the center of the notch along the edge of the specimen and the length of the pre-crack was re-measured.

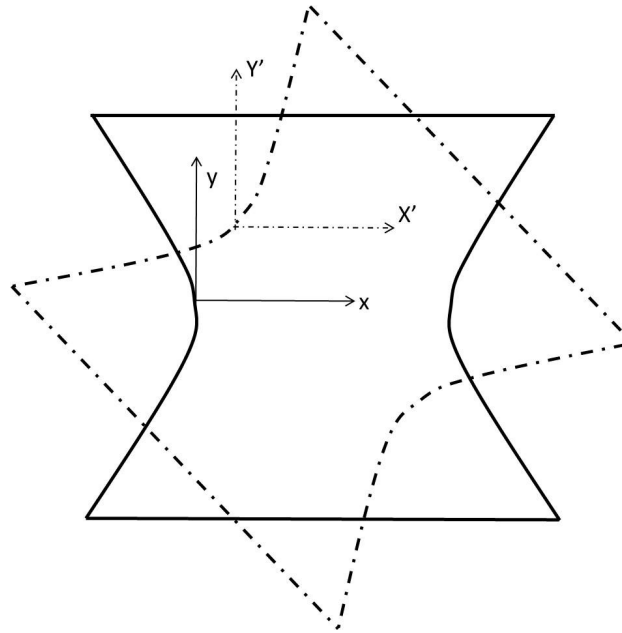


Figure 2.5. Schematic of coordinate system in which the crack tip was tracked during pre-cracking (solid line) and testing (dotted line)

Following the procedure outlined in the previous steps, a total of 6 experiments were performed at loading angles  $\Phi = 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ,$  and  $90^\circ$ , with  $\Phi = 90^\circ$  degrees being nominally Mode II crack loading. For the loading cases  $\Phi = 15^\circ, 30^\circ,$  and  $45^\circ$ , the one degree of freedom slide apparatus was used for tracking the crack tip. The two degree of freedom slide apparatus was built and used to track the crack tip for  $\Phi = 60^\circ, 75^\circ,$  and  $90^\circ$ . Again, fatigue loading at 10Hz was applied in force control according to the loads outlined in the following section. The crack tip position was measured approximately every 5,000 to 20,000 cycles.

## 2.4 LOAD PREDICTION

Load data was predicted for fatigue pre-cracking and testing based on the load predictions for tests performed previously for NASA Langley Research Center and the US Air Force [21]. Load shedding was performed to avoid the risk of initiating stable tearing or formation of a large plastic zone at the crack tip. The loading ratio,  $R$ , and the amplitude of  $\Delta K_I$  were held constant at 0.17 and  $359 \text{ Pa}\cdot\text{m}^{1/2}$  respectively, by allowing the load to decrease as the crack length increased.

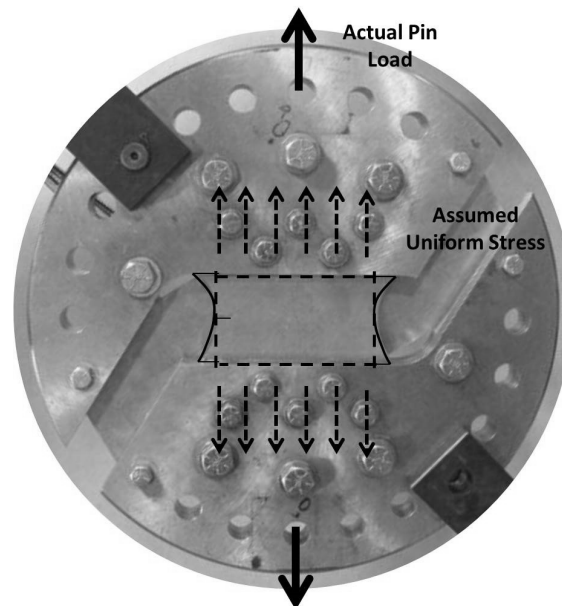


Figure 2.6. Visual representation of the actual geometry and loading (solid lines) and the assumed geometry and loading (dotted lines).

The SIF was estimated using an empirical expression from Tada [22] that is valid for a through-thickness edge crack under uniform uniaxial tension. It was assumed that the width was the transverse dimension of the butterfly specimen at its smallest cross-section, which is also where the notch is located. As shown schematically in Figures 2.6 and 2.7,

an estimate for the uniform applied stress was obtained using the load applied at the pin divided by the cross-sectional area of the specimen using the assumed width and actual thickness,  $w = 152.4$  mm and  $t = 6.35$  mm.

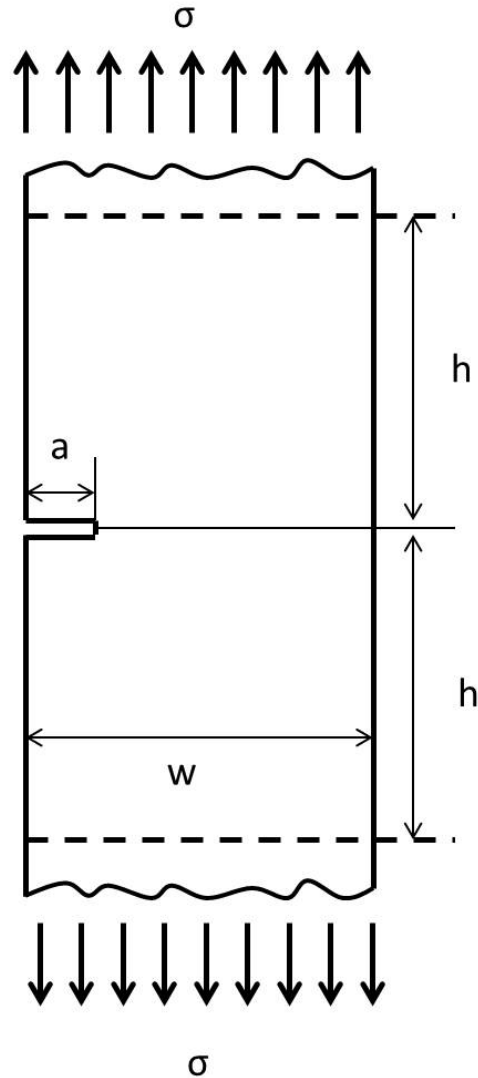


Figure 2.7. Diagram of geometry for Tada's empirical expression.

$$f\left(\frac{a}{w}\right) = \sqrt{\frac{2w}{\pi a} \tan \frac{\pi a}{2w}} * \frac{0.752 + 2.02(a/w) + 0.37(1 - \sin \frac{\pi a}{2w})^3}{\cos \frac{\pi a}{2w}} \quad [2.1]$$

Using Eqs 1.1, 1.2 and 2.1 [22],  $\Delta K$  was estimated.

While performing the first experiment, which was for the 15° loading case, it was observed that the crack path on the front of the specimen deviated from the crack path measured on the back surface after a few millimeters of crack extension. These observations led to the implementation of a different approach for load prediction to avoid unusual crack propagation in future experiments. The goal of the modified approach was to keep  $\Delta K$  constant in order to avoid excessive plasticity in the crack tip region, crack slanting, or crack tearing. Since the method for estimating the SIF was quite crude, and did not account for the various loading angles and resulting  $K_I$  and  $K_{II}$  values, it was determined that following Paris' Law for the material was a more accurate method of crack growth control for  $\Delta K_{eq}$  which is defined as follows [23];

$$\Delta K_{eq} = \gamma \Delta K_I + (1 - \gamma) \sqrt{(\Delta K_I)^2 + \gamma_1 (\Delta K_{II})^2 + \gamma_2 (\Delta K_{III})^2} \quad [2.2]$$

where  $\gamma$ ,  $\gamma_1$ , and  $\gamma_2$  are parameters to be defined. Using  $\Delta K_{eq}$  and assuming that there is no crack closure effect, the crack growth rate can be determined using Eq 1.5. That is, the authors opted to maintain the same crack growth rate throughout the experiment.

For the next experiment, the 30° loading case, pre-cracking was performed according to the loads originally predicted. However after the specimen was rotated, the new method of determining the loading was performed. From the previous test data, it was determined that a crack growth rate of  $\approx 6 \times 10^{-5}$  mm/cycle was a safe rate to run the experiments and maintain nominally linear elastic conditions.. A loading ratio  $R = 0.4$  was chosen for the experiment. This crack growth rate was maintained by allowing the crack to grow until the rate increased to  $\approx 8 \times 10^{-5}$  mm/cycle. The load was then dropped by approximately 5%, resulting in a crack growth rate of  $\approx 4 \times 10^{-5}$  mm/cycle. This

process was repeated to maintain an average crack growth rate of  $\approx 6 \times 10^{-5}$  mm/cycle and therefore maintain a constant average  $\Delta K_{eq}$  during the experiment.

For loading angles  $45^\circ$ ,  $60^\circ$ ,  $75^\circ$ , and  $90^\circ$ , the new pre-cracking loads and method of load shedding to control crack growth rate were recomputed to maintain an approximately constant crack growth rate. In all the remaining experiments,  $R = 0.4$ . Cycle count, crack growth, and load data for each experiment are provided in Appendix A. Recall for experiments for  $\Phi = 15^\circ$ ,  $30^\circ$ , and  $45^\circ$  only the one degree of freedom horizontal uni-slide was used to visually track the crack so the recorded value in the appendix is only the x position as shown in Figure 2.5. For  $\Phi = 60^\circ$ ,  $x'$  and  $y'$  positions are recorded and reported in the appendix. Recorded data for  $\Phi = 75^\circ$  is not reported since the crack did not propagate after applying hundreds of thousands of load cycles using the same loads as applied in the  $\Phi = 60^\circ$  experiment.

## 2.5 DETERMINATION OF EXPERIMENTAL CRACK PATHS

In order to obtain the experimental crack path for each loading case, images of the front and back of each specimen were necessary after the fatigue tests were conducted. The surfaces of each of the specimens had to be prepared so that when images were obtained, the crack would be visible and there would be no reflection on the surface. First, the surface was sanded with 600 grit sand paper to roughen the surface and remove the mirror finish. Then dry pigment was rubbed into the crack. The surface was sanded again to remove excess pigment on the surface, leaving the remaining pigment in the crack.

A 2.5 Megapixel Point Grey CCD camera was positioned on a tripod to ensure that images were taken perpendicular to the surface of the specimen. The specimen was positioned, and two rulers were placed on the surface of the specimen – one vertically and the other horizontally-- to provide a scale when the images were digitized. Images were taken of the front and back surfaces, and loaded into GetData Graph Digitizer software [24]. First, using the ruler, 5 points were created at position (X,Y) and 5 more points were created at (X+1", Y+1") as shown in Figure 2.8.

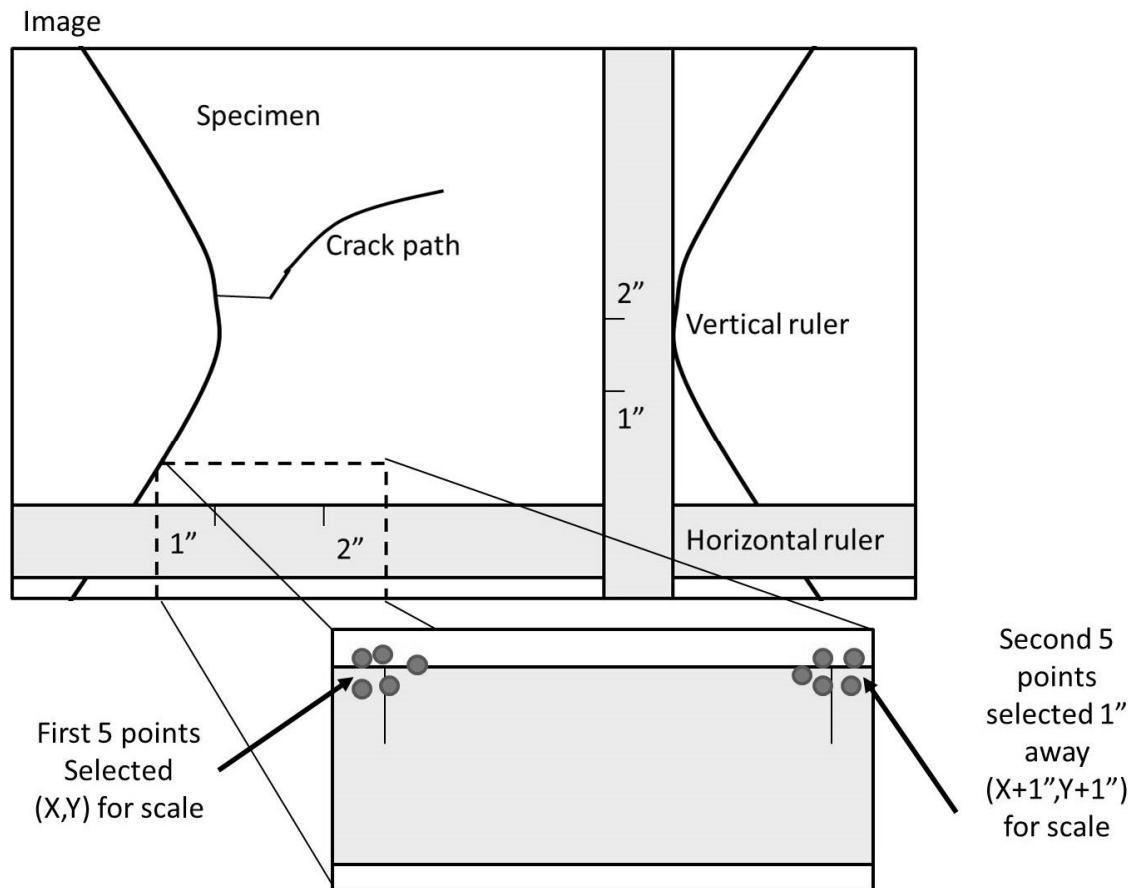


Figure 2.8. Schematic of the images digitized and an exaggerated view of how the points were selected for determining the scale factor from pixels to meters.

The distance between each set of points for the front (*f*) and back (*b*) of specimens for each loading case,  $\Phi$ , were determined in pixels and averaged to be used for a scale factor. For each specimen, the average value and the standard deviation of the vertical and horizontal scale factors for each case for the front (back) ,  $s_{v,f}^{\Phi} \pm \Delta s_{v,f}^{\Phi}$  ( $s_{v,b}^{\Phi} \pm \Delta s_{v,b}^{\Phi}$ ) and  $s_{h,f}^{\Phi} \pm \Delta s_{h,f}^{\Phi}$  ( $s_{h,b}^{\Phi} \pm \Delta s_{h,b}^{\Phi}$ ) respectively, are given in Table 2.1. Then *i* points were selected along the front and back crack paths for each loading case,  $(x_i^{\Phi}, y_i^{\Phi})_f$  and  $(x_i^{\Phi}, y_i^{\Phi})_b$ , and exported to Microsoft Excel. It was assumed that the error associated with selecting the points along the crack path was  $\Delta x_i^{\Phi}, \Delta y_i^{\Phi} \approx \pm 1$  pixel. Points along the path were converted from pixels to meters using the scale factor, to give metric positions  $(X_i^{\Phi}, Y_i^{\Phi})_f$  and  $(X_i^{\Phi}, Y_i^{\Phi})_b$ .

$$X_i^{\Phi} = s_h^{\Phi} * x_i^{\Phi}, Y_i^{\Phi} = s_v^{\Phi} * y_i^{\Phi} \quad [2.3]$$

The standard deviation for the front and back of each specimen associated with the crack path position,  $(X_i^{\Phi}, Y_i^{\Phi})_f$  and  $(X_i^{\Phi}, Y_i^{\Phi})_b$ , was determined using error propagation for multiplication [25]:

$$\begin{aligned} \Delta X_{i,f}^{\Phi} &= X_{i,f}^{\Phi} * \sqrt{\left(\frac{\Delta s_{h,f}^{\Phi}}{s_{h,f}^{\Phi}}\right)^2 + \left(\frac{\Delta x_i^{\Phi}}{x_{i,f}^{\Phi}}\right)^2}, \Delta Y_{i,f}^{\Phi} = Y_{i,f}^{\Phi} * \sqrt{\left(\frac{\Delta s_{v,f}^{\Phi}}{s_{v,f}^{\Phi}}\right)^2 + \left(\frac{\Delta y_i^{\Phi}}{y_{i,f}^{\Phi}}\right)^2} \\ \Delta X_{i,b}^{\Phi} &= X_{i,b}^{\Phi} * \sqrt{\left(\frac{\Delta s_{h,b}^{\Phi}}{s_{h,b}^{\Phi}}\right)^2 + \left(\frac{\Delta x_i^{\Phi}}{x_{i,b}^{\Phi}}\right)^2}, \Delta Y_{i,b}^{\Phi} = Y_{i,b}^{\Phi} * \sqrt{\left(\frac{\Delta s_{v,b}^{\Phi}}{s_{v,b}^{\Phi}}\right)^2 + \left(\frac{\Delta y_i^{\Phi}}{y_{i,b}^{\Phi}}\right)^2} \end{aligned} \quad [2.4]$$

The percent error in the crack path for each loading case,  $\Phi$ , is defined as the maximum percent error of the  $X_i^{\Phi}$  and  $Y_i^{\Phi}$  points on the front and back of the specimen and is as follows:



$$\text{Percent Error}^{\phi} = 2 * \left\langle \frac{\Delta X_{i,f}^{\phi}}{X_{i,f}^{\phi}}, \frac{\Delta X_{i,b}^{\phi}}{X_{i,b}^{\phi}}, \frac{\Delta Y_{i,f}^{\phi}}{Y_{i,f}^{\phi}}, \frac{\Delta Y_{i,b}^{\phi}}{Y_{i,b}^{\phi}} \right\rangle_{max} * 100 \quad [2.5]$$

and are reported in Table 2.2. As shown in Table 2.2, the estimated errors are small and assumed to be negligible. From this point forward only the average crack path points will be considered in analysis.

Using Microsoft PowerPoint, the images for the front and back of each specimen were set to  $\approx 50\%$  transparency and layered on top of each other. It was observed that the specimen was slightly rotated in a couple of those images. The images were rotated such that the edges of the specimen in both images were aligned. Using the angle of rotation used in Power Point to align the images, the crack paths corresponding to those images were rotated by the same angle. Then, the average points for each specimen were translated to the coordinate system shown in Figure 2.9. The X and Y points along the crack for the front and back of each specimen were plotted, and a second order polynomial was fitted to the data using least squares. To verify that the digitized crack path was accurate, a plot of the polynomial fit was layered on top of the image of the specimen.

Table 2.1. Scale factors used to convert digitized points from pixels to meters.

Loading Case	Front Image Scale Factor (m/pixel x 10 <sup>-5</sup> )		Back Image Scale Factor (m/pixel x 10 <sup>-5</sup> )	
	Vertical Scale	Horizontal Scale	Vertical Scale	Horizontal Scale
15°	6.44 ± 0.03	6.41 ± 0.02	6.43 ± 0.02	6.39 ± 0.01
30°	7.14 ± 0.05	7.09 ± 0.04	7.14 ± 0.06	7.13 ± 0.03
45°	6.50 ± 0.07	6.48 ± 0.02	6.56 ± 0.02	6.52 ± 0.02
60°	6.48 ± 0.02	6.51 ± 0.00	6.55 ± 0.01	6.52 ± 0.03

Table 2.2. Percent error in crack path position.

Loading Case	Percent Error
15°	1.0%
30°	1.3%
45°	2.2%
60°	0.8%

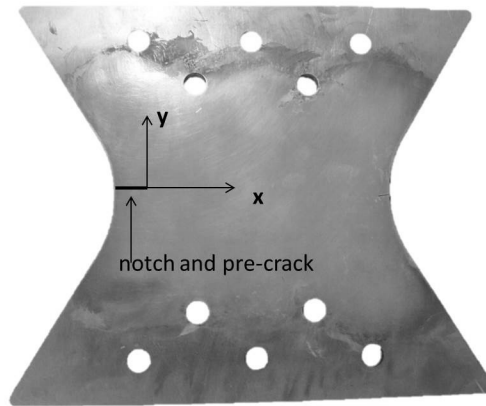


Figure 2.9. Coordinate system used to define crack paths from the tip of the pre-crack.

For the cases where the two degree of freedom slide apparatus was used, the x and y data points recorded during the experiment in the coordinate system shown in Figure 2.5 were rotated and translated into the coordinate system in Figure 2.9, in addition to digitizing the crack path.

## 2.6 EXPERIMENTAL RESULTS

For the experiments performed using the modified load prediction method of controlling the crack growth rate, the discrete crack growth rate  $\Delta a/\Delta N$  was plotted along the crack length  $a$  in Figures 2.10-2.12.

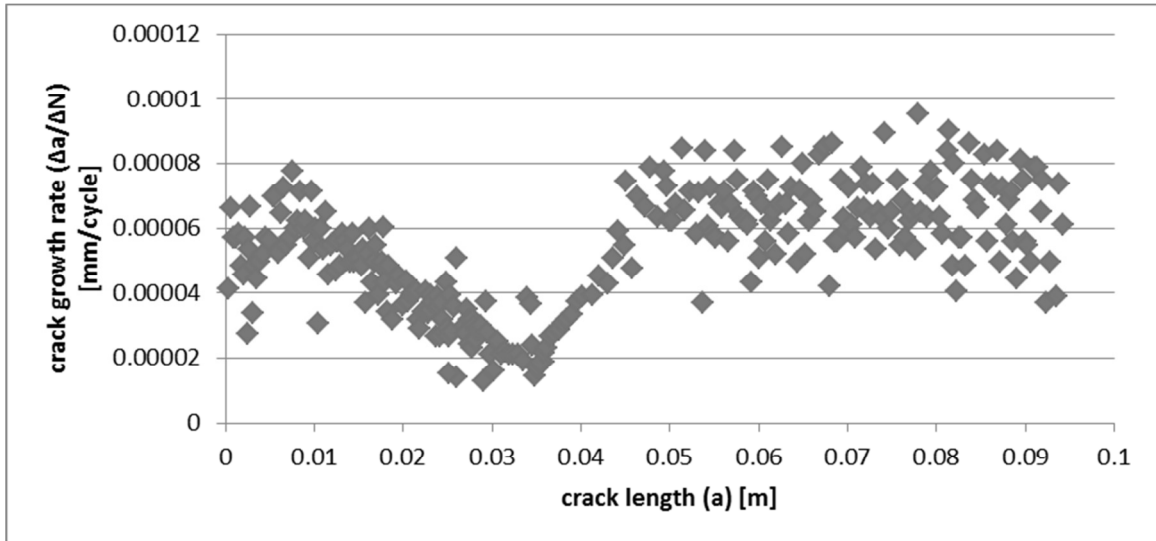


Figure 2.10. Crack growth rate along crack path for 30° loading case.

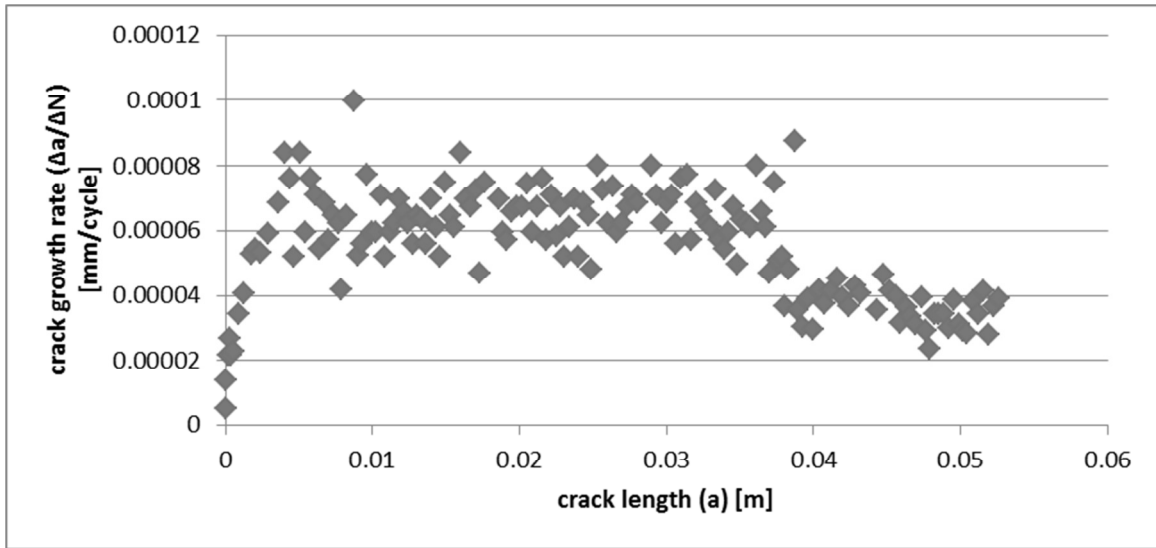


Figure 2.11. Crack growth rate along crack path for 45° loading case.

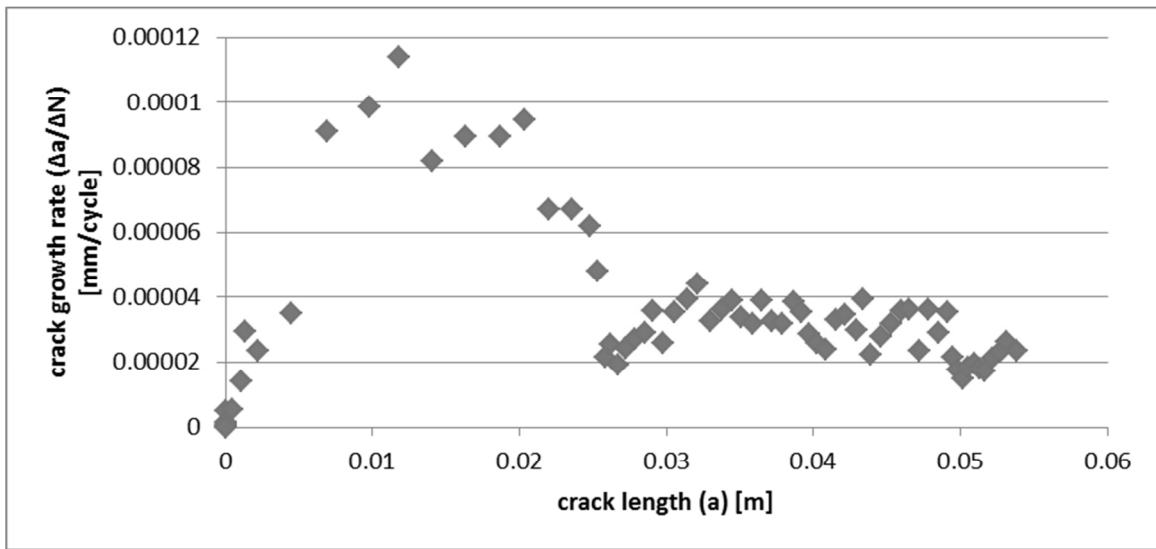


Figure 2.12. Crack growth rate along crack path for 60° loading case.

For loading cases 15°, 30°, 45°, and 60°, fatigue crack propagation occurred, and for loading cases 75° and 90°, no crack propagation occurred. Figure 2.13 shows the originally digitized crack path for the 15° loading case. Figure 2.14 shows the data for the 15° loading case crack path before crack slanting occurred along with the polynomial fit for the data. Figures 2.15-2.17 show the digitized crack path data for the front and back

of the specimens for the 30°, 45°, and 60° loading cases respectively along with the polynomial fit for each data set.

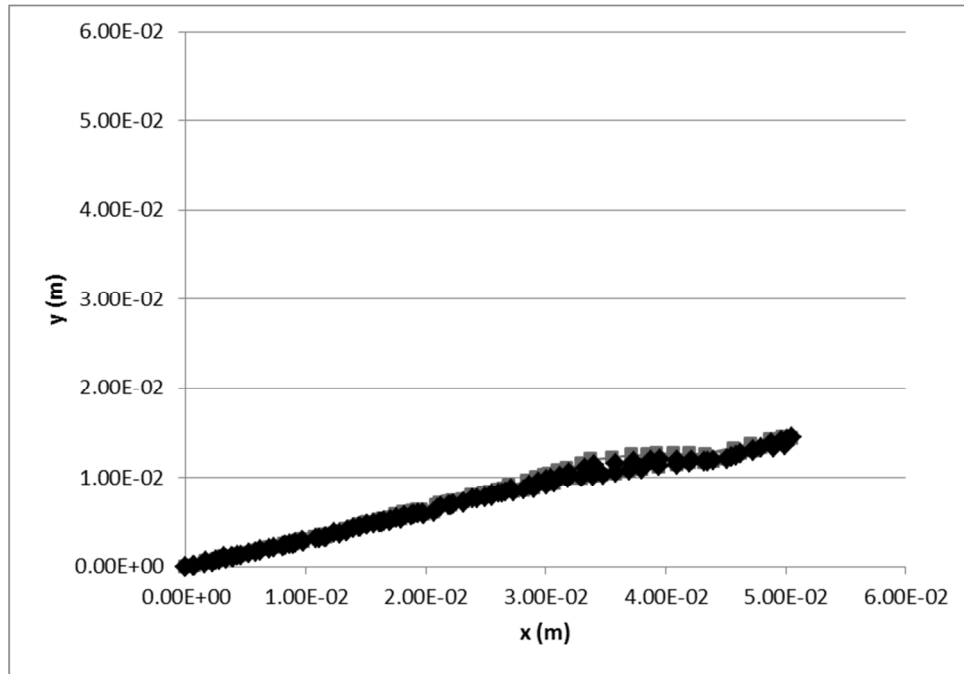


Figure 2.13. Originally digitized crack path data for 15° loading case.

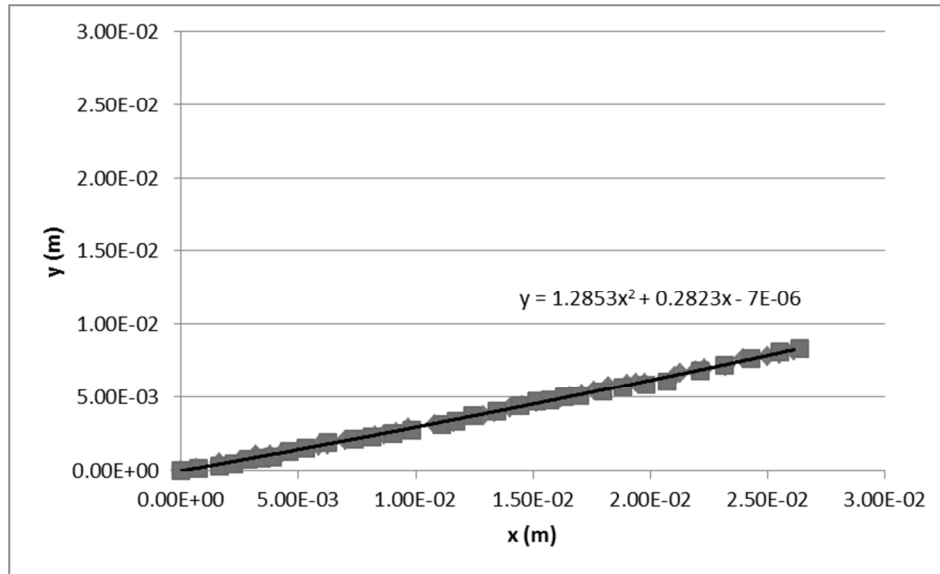


Figure 2.14. Digitized crack path and polynomial fit for 15° loading case.

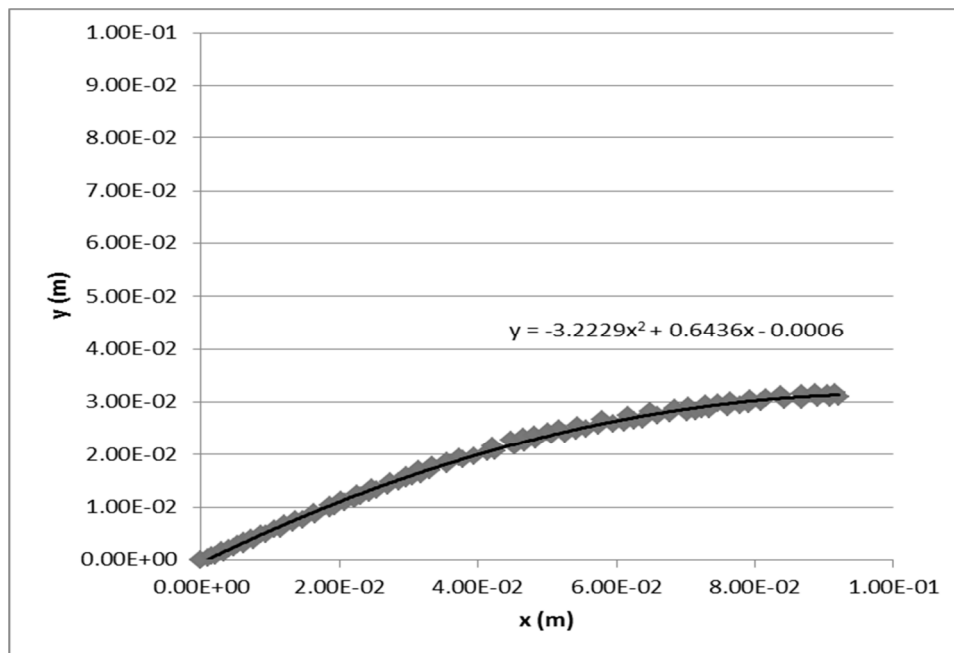


Figure 2.15. Digitized crack path and polynomial fit for 30° loading case.

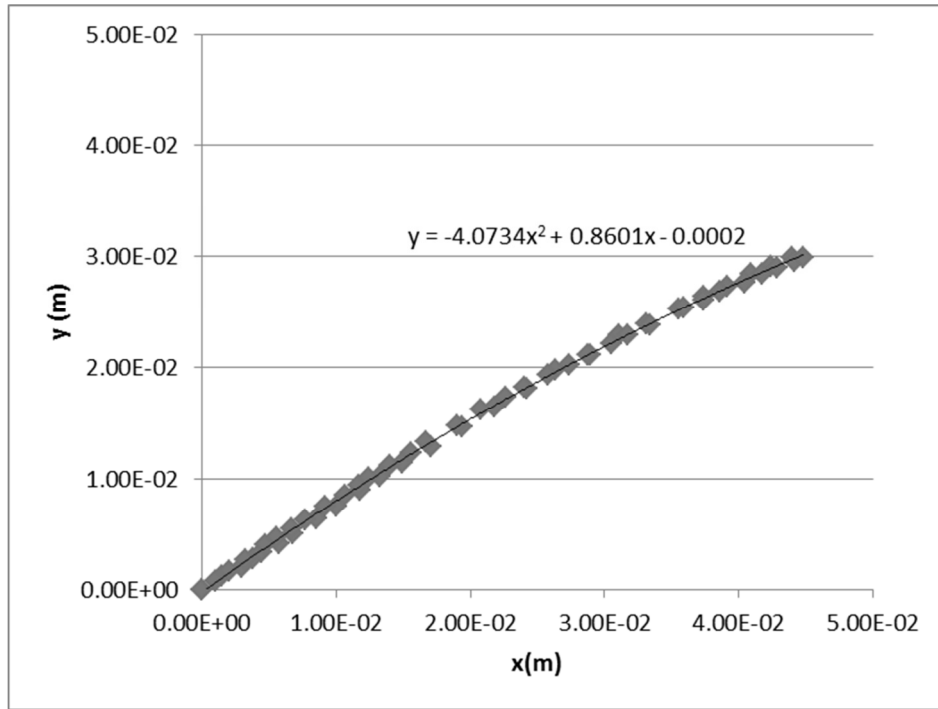


Figure 2.16. Digitized crack path and polynomial fit for 45° loading case.

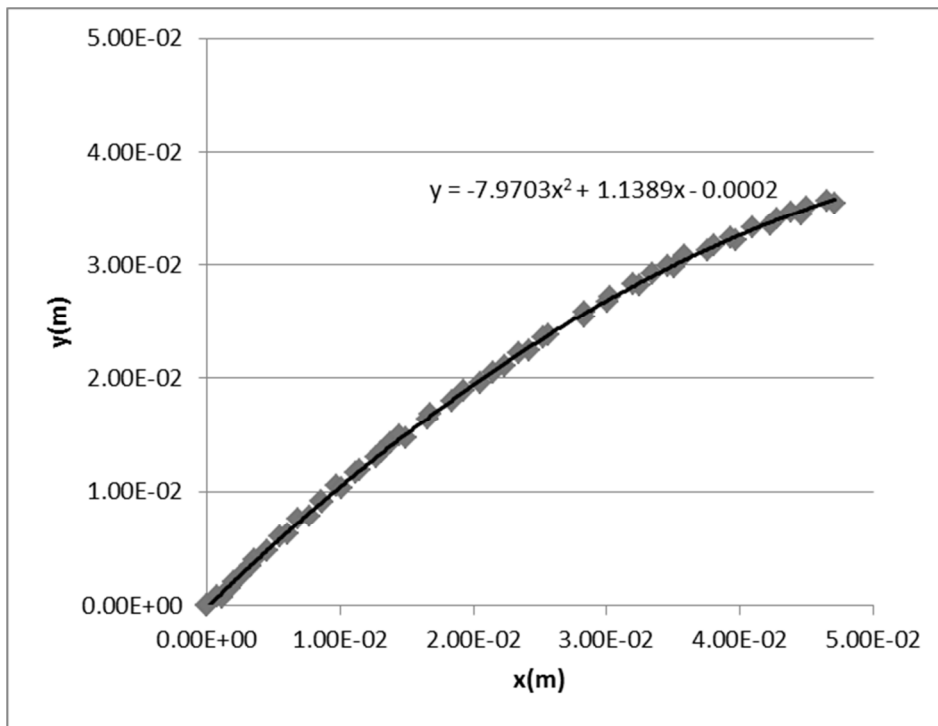


Figure 2.17. Digitized crack path and polynomial fit for 60° loading case.

The rotated and translated crack path determined experimentally using the two degree of freedom slide apparatus and the digitized crack path for  $\Phi = 60^\circ$  were plotted in Figure 2.18 to verify the accuracy of the digitization process. The following plot shows that the two methods of obtaining the experimental crack path are in good agreement with each other. The calipers used for measuring the amount of travel of the slide and microscope objective have an accuracy of 0.00127mm which results in less than 0.01% error in the measuring process. Even though the digitized crack path for  $\Phi = 60^\circ$  had 0.8% error (Table 2.2), the digitization process has more opportunity to induce error through obtaining, aligning, digitizing, and scaling the images. While error in either path is negligible, the process of directly measuring the crack tip location using the dual caliper apparatus during the experiment is accurate, efficient, and has less opportunity for inducing error.

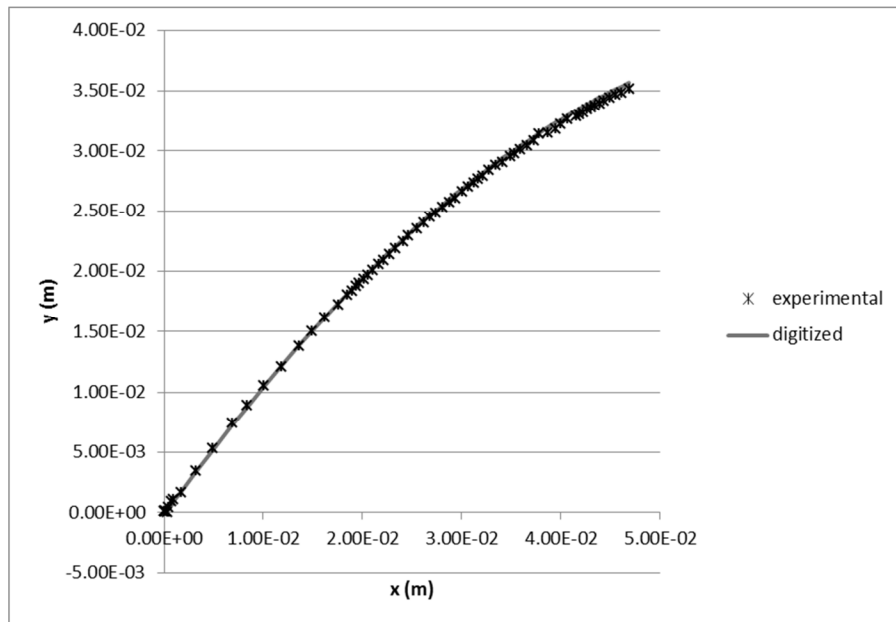


Figure 2.18. Crack path for  $\Phi = 60^\circ$  determined by slide apparatus experimentally versus the crack path determined by digitization.



## CHAPTER 3

### THEORETICAL WORK

#### 3.1 CRACK3D

CRACK3D is a three-dimensional finite element code first developed by the University of South Carolina and later jointly by the University of South Carolina and Correlated Solutions, Inc.. It is capable of simulating elastic-plastic stable tearing crack extension and linear-elastic fatigue crack propagation, both with curved crack fronts and curvilinear crack paths for mixed-mode conditions. Two methods of crack growth simulations are available: nodal release and local re-meshing. Nodal release assumes that the crack path is known prior to running the simulation and is useful in evaluating crack growth events with known crack paths from experimental measurements or for what-if design scenarios. In the case that the crack path is to be predicted, local re-meshing is used to extend the crack [1] [2] [3]. For the case of fatigue crack propagation, there are three steps to crack growth predictions: (1) will the crack grow? (2) in what direction will it grow? (3) how far will it extend for a certain number of loading cycles or how many loading cycles will be required to extend the crack by a certain amount?

For determining if the crack will propagate,  $\Delta K > \Delta K_{TH}$  must be true as discussed in Section 1.2. CRACK3D can be used to evaluate  $\Delta K$ , which can be used to check if  $\Delta K > \Delta K_{TH}$  is satisfied. Once this crack growth criterion is met, CRACK3D can be used to simulate the crack growth process and predict (a) the direction of crack growth and (b)

the variations of stress intensity factors with the amount of crack growth, which can be used to predict the number of loading cycles as a function of the amount of crack growth.

In CRACK3D the determination of stress intensity factors is done using the method of three-dimensional virtual crack closure technique (3D-VCCT) [3] [1] [2] [4] [5], which is based on the approach of the strain energy release rate [6], which is the amount of energy released per unit thickness per unit crack extension when new crack surfaces are created during crack extension. The 3D-VCCT can be used in finite element simulations to calculate accurately and efficiently the mixed-mode strain energy release rates,  $G_I$ ,  $G_{II}$ , and  $G_{III}$ , which are related to the mixed-mode stress intensity factors  $K_I$ ,  $K_{II}$ , and  $K_{III}$ . Since fatigue crack propagation often occurs under nominally linearly elastic conditions, it is assumed that the amount of energy required to extend the crack a small increment is the same as the amount of energy required to close the crack. In Mode I, the work required to close the crack per unit thickness is equivalent to one half the nodal force multiplied with the opening displacement. In VCCT, this product between the nodal force and opening displacement is approximated by using the nodal force at nodes immediately ahead of the crack front and the crack opening displacement at corresponding nodes immediately behind the crack front from the same finite element solution. Okada et al. [4] applied VCCT to three-dimensional analysis using tetrahedral elements. Deng et al. [3] [1] [2] later adopted Okada's 3D-VCCT for general crack growth simulations by proposing a locally structured re-meshing approach.

To illustrate the 3D-VCCT, consider crack growth simulations using the nodal release option. The extended crack is created by separating the crack front and mid-nodes attached to the element just ahead of the crack front into coincident nodal pairs. It is assumed that these nodal pairs are connected with a stiff spring with length zero as shown in Figure 3.1.

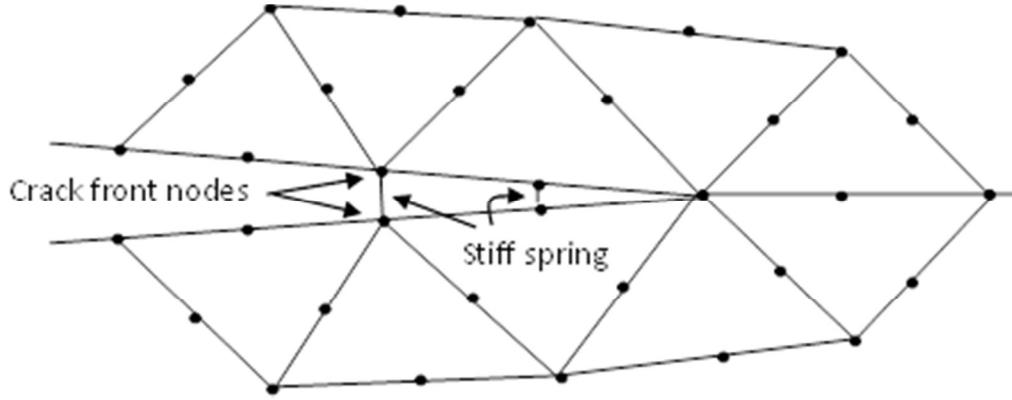


Figure 3.1. An exaggerated local 2D view of a crack-front finite element mesh on the plane normal to the crack front, where the rigid springs between the node pairs have zero length.

Stiff spring constants,  $K_x$ ,  $K_y$ , and  $K_z$ , which are large but otherwise arbitrary values set in the nodal release option, and the displacements,  $u_x$ ,  $u_y$ , and  $u_z$ , of the upper (+) and lower (-) nodes are used to compute the forces,  $F_x$ ,  $F_y$ , and  $F_z$  (Eq 3.1) for each node, where the coordinate system is such that  $x$  is along the direction of crack extension,  $y$  is perpendicular to crack extension, and  $z$  is through the thickness and tangent to the crack front.

$$F_x = K_x(u_x^+ - u_x^-), F_y = K_y(u_y^+ - u_y^-), F_z = K_z(u_z^+ - u_z^-) \quad [3.1]$$

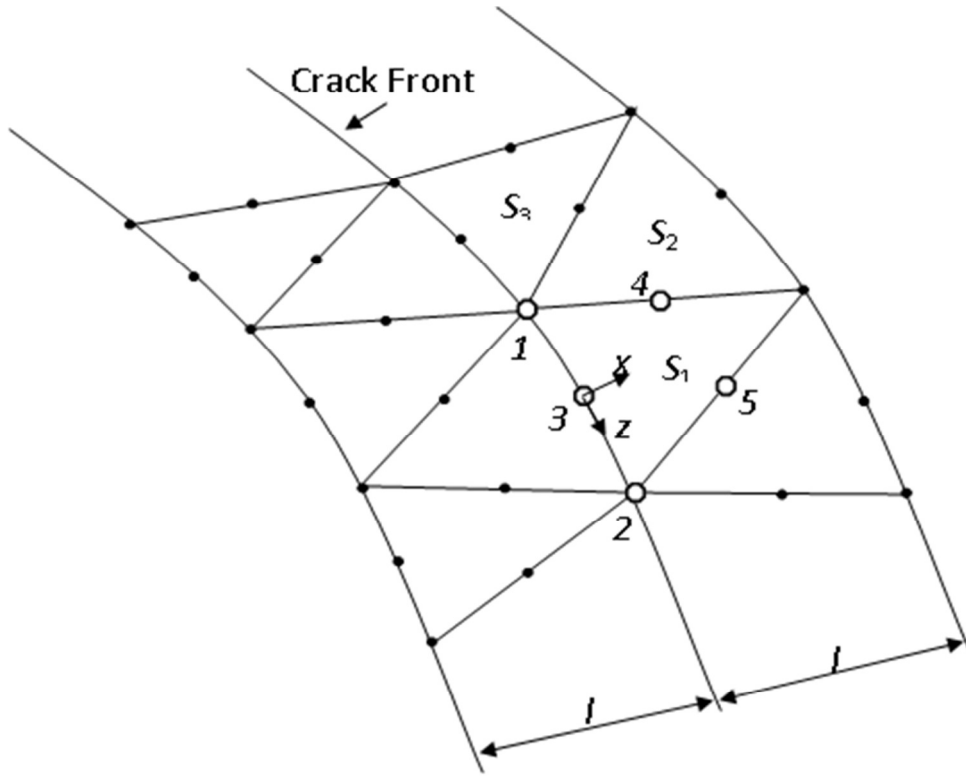


Figure 3.2. A local view of a crack front mesh on the extended crack surface, where the local coordinate system for a mid-node on the crack front has its origin at the node.

Figure 3.2 shows the view of the crack front through the thickness where  $l$  is one element length and elements 1, 2, and 3 only.  $S_1$ ,  $S_2$ , and  $S_3$  are the areas of the sides of the tetrahedral elements on the extended crack surface for elements 1, 2, and 3 respectively. Nodes 1, 2, and 3 are nodes located on the crack front where node 1 is attached to elements 1, 2, and 3 and node 2 is only attached to element 1.

Some nodes, such as 1 in Fig. 3.2, share element surfaces therefore the resultant forces,  $F_x$ ,  $F_y$ , and  $F_z$  for node 1, must be divided among the surfaces  $S_1$ ,  $S_2$ , and  $S_3$ . For element surface 1,

$$F_{x1} = \frac{S_1 F_x}{S_1 + S_2 + S_3}, F_{y1} = \frac{S_1 F_y}{S_1 + S_2 + S_3}, F_{z1} = \frac{S_1 F_z}{S_1 + S_2 + S_3} \quad [3.2]$$

Then for element surface 1, the 3-D strain energy release rates,  $G_I$ ,  $G_{II}$ , and  $G_{III}$ , are estimated by summing up the work required to close the nodal pairs on the element surface and are expressed as [4]

$$G_I \approx \frac{1}{3S_1} \sum_i F_{yi} u_{yi}, G_{II} \approx \frac{1}{3S_1} \sum_i F_{xi} u_{xi}, G_{III} \approx \frac{1}{3S_1} \sum_i F_{zi} u_{zi} \quad [3.3]$$

where  $u_{xi}$ ,  $u_{yi}$ , and  $u_{zi}$  are the relative displacements between the top and bottom crack surfaces at nodes behind the crack front that correspond to nodal forces  $F_{xi}$ ,  $F_{yi}$ , and  $F_{zi}$  for  $i$  nodes attached to the element surface ahead of the crack front. Finally, the SIFs for plane strain are related to strain energy release rates by

$$K_I = \sqrt{\frac{G_I E}{1-\nu^2}}, K_{II} = \pm \sqrt{\frac{G_{II} E}{1-\nu^2}}, K_{III} = \pm \sqrt{\frac{2G_{III} E}{2(1+\nu)}} \quad [3.4]$$

where  $E$  is Young's modulus and  $\nu$  is Poisson's ratio. It is noted that the signs for  $K_{II}$  and  $K_{III}$  are the same as the signs of the relative displacements behind the crack front along the  $x$  and  $z$  axes, respectively.

It is noted that the SIF values described above correspond to the maximum loading value applied during a loading cycle. Once the SIFs for the maximum applied load are predicted using the VCCT, the direction in which the crack will propagate is predicted using MCS criterion [5]. The MCS criterion assumes that a crack will grow in the direction,  $\theta_c$ , that maximizes the local circumferential stress,  $\sigma_{\theta\theta}$ , at a specified location ahead of the crack tip. The local stress around the crack tip can be expressed as a function of SIF and position with respect to the crack tip in polar coordinates  $(r, \theta)$  (see Figure 3.3).

$$\begin{aligned} \sigma_{\theta\theta} &= \frac{1}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left( K_I \left(\cos\left(\frac{\theta}{2}\right)\right)^2 - \frac{3}{2} K_{II} \sin\theta \right) \\ \sigma_{r\theta} &= \frac{1}{2\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left( K_I \sin\theta + K_{II} (3 \cos\theta - 1) \right) \end{aligned} \quad [3.5]$$

Where  $\sigma_{\theta\theta}$  is the circumferential normal stress near the crack tip and  $\sigma_{r\theta}$  is the shear stress.

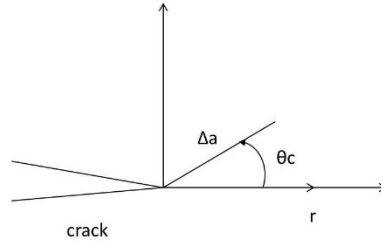


Figure 3.3. Diagram of crack growth direction according to the MCS criterion.

It can be shown that that  $\sigma_{\theta\theta}$  is maximum or minimum when the shear stress  $\sigma_{r\theta}$  is zero.

So setting  $\sigma_{r\theta}$  from Eq 3.5 to zero

$$0 = (K_I \sin \theta + K_{II}(3 \cos \theta - 1)) \quad [3.6]$$

and solving for  $\theta$ ,

$$\theta_c = 2 \tan^{-1} \left( \frac{\frac{K_I}{K_{II}} \pm \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}}{4} \right) \quad [3.7]$$

The root that maximizes  $\sigma_{\theta\theta}$  gives  $\theta_c$  as the direction in which the crack will extend [5].

To apply the VCCT in crack growth simulations using the local re-meshing option (instead of the nodal release option), the local mesh immediately ahead and behind the crack front must be properly structured, so that the local mesh immediately behind the crack front can be viewed as being shifted by one element size from the local mesh immediately ahead of the crack front, Therefore, once crack growth is determined to occur along a certain direction with a certain increment, the new mesh around the new crack front is generated such that there is a structured mesh (within a local re-meshing

zone around the new crack front) with equal number of elements behind the crack front and ahead of the crack front. [3] [1] [2]

A fatigue crack growth rate model, such as the Paris' Law, is used to determine how many cycles it will take for the crack to grow the amount of crack extension chosen by the user [7]. However, since mixed-mode conditions are considered,  $\Delta K$  as presented in Chapter 1 is no longer Mode I. After  $K_I$ ,  $K_{II}$ , and  $K_{III}$  are predicted,  $\Delta K_I$ ,  $\Delta K_{II}$ , and  $\Delta K_{III}$  can be computed using Eq 1.2, and  $\Delta K_{eq}$  from Eq 2.2. Again, assuming that there is no crack closure effect, the crack growth rate can be determined using a Paris-type Law as in Eq 1.5.

### 3.2 GEOMETRY, MESH GENERATION, AND BOUNDARY CONDITIONS

The Arcan fixture and specimen were modeled as shown in Figure 3.4. The fixture and specimen are connected by bolts. For simplicity, this fixture-specimen connection is approximated by a continuous bond at the fixture-specimen boundary. To this end, the bolts are not modeled and the fixture and specimen are treated as three solid regions with different thicknesses. Also, the outside radius of the fixture in the model corresponds to the radius of the center of the pin holes on the actual fixture.

An idealized through-thickness edge notch and pre-crack exactly 12.7mm long was modeled as the initial crack in the exact geometric vertical center of the specimen and is perfectly horizontal into the width of the specimen. Material properties for the fixture and specimen are Young's modulus =  $2.07 \times 10^{11}$  Pa and  $7.11 \times 10^{10}$  Pa and Poisson's ratio = 0.30 and 0.33 respectively. Both materials are modeled as being elastic-plastic through the use of their actual stress-strain curves.

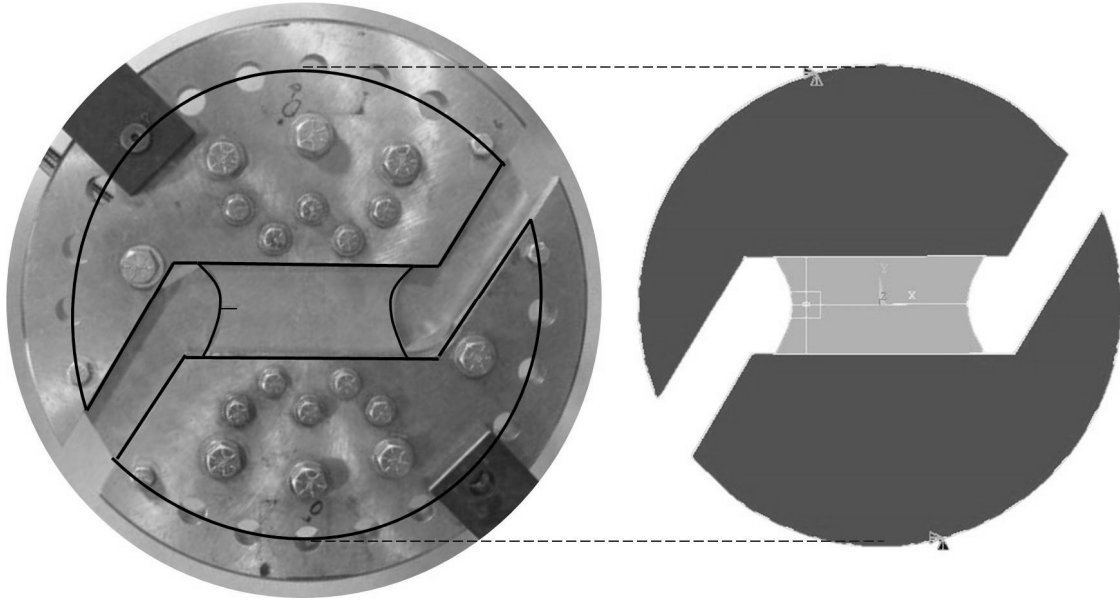


Figure 3.4. Diagram of a picture of actual Arcan fixture and specimen (left) and image of finite element model geometry (right).

The volumes were then meshed with 10 noded tetrahedral elements. Figure 3.5 shows the initial mesh generated, and Figure 3.6 shows the initial three zones of the mesh. As discussed in Section 3.1 for local re-meshing, structured elements were created one element ahead and one element behind the crack front. Then a transition zone was created from the structured element to the far field mesh. Elements in this zone should transition from the local minimum element size to a maximum element size at the boundary of the local re-meshing zone. The far-field mesh away from the local region can be coarse, provided it can adequately transfer loading information to the crack front local region.



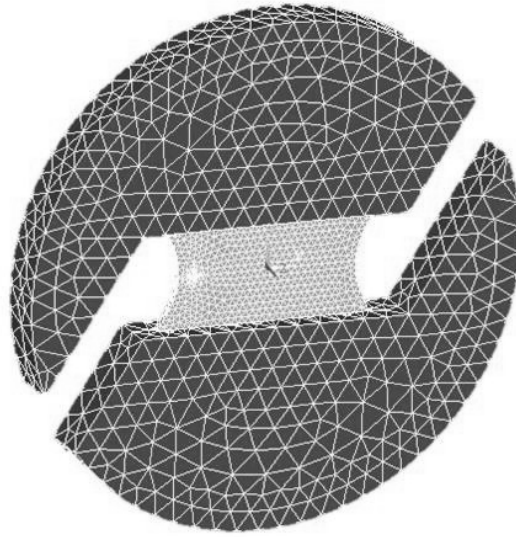


Figure 3.5. Image of the 3D mesh.

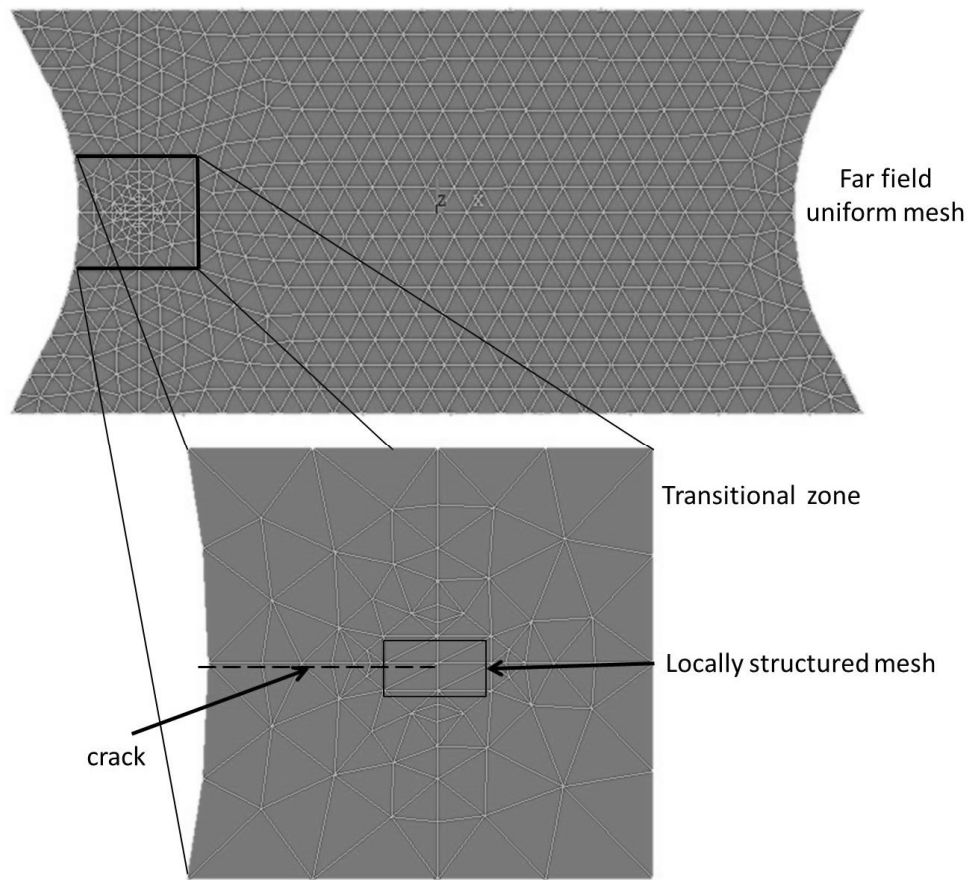


Figure 3.6. A 2D view of the initial 3D finite element mesh in the specimen region.

The first mesh shown in Figure 3.6 has two elements through the thickness. A second refined mesh was generated with 4 elements through the thickness in the structured area to check for convergence of the mesh.

For each loading angle,  $\Phi$ , a set of lines (one on the top fixture and the other on the bottom fixture) corresponding to the center of the pins were created on the surface of the fixture model in the through-thickness direction. The boundary conditions were such that the displacement of the bottom line was set to zero in the x and y directions ( $u_x, & u_y = 0$ ) and only the z displacement specified was of the center point on the bottom line ( $u_z = 0$ ). The displacement of the corresponding top line had a magnitude of  $1 \times 10^{-3}$  mm along the direction of loading,  $\Phi$ , as shown in Figure 3.7, which was decomposed into x and y components (see Table 3.1).

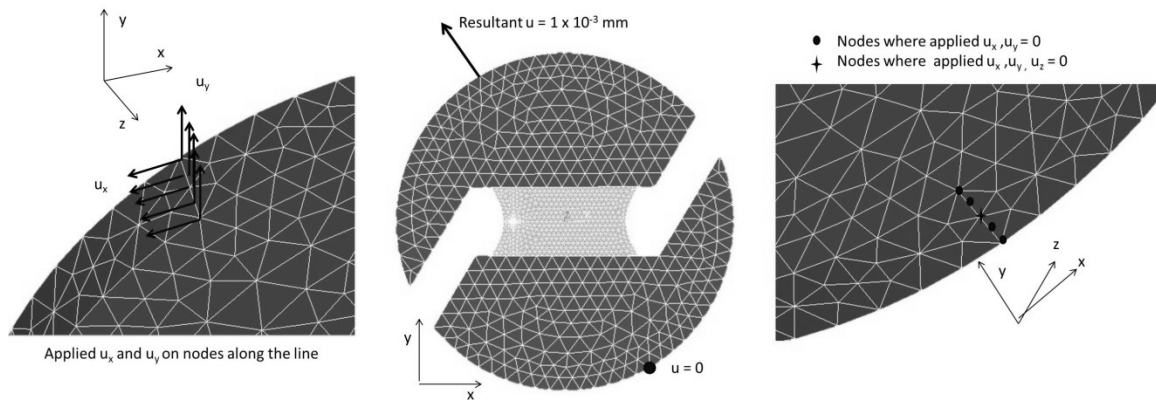


Figure 3.7. Boundary conditions at  $\Phi = 30^\circ$ .

Table 3.1. Table of applied displacements for line on top fixture.

Loading Case	Displacement in x direction ( $u_x$ ) [ $1 \times 10^{-3}$ mm]	Displacement in y direction ( $u_y$ ) [ $1 \times 10^{-3}$ mm]
15°	-0.259	0.966
30°	-0.500	0.866
45°	-0.707	0.707
60°	-0.866	0.500

### 3.3 SIMULATION PROCEDURE

The initial finite element meshes were created using ANSYS 14 [9]. Mesh data for both the initial and initial refined meshes was exported from ANSYS using the “NWRITE” and “EWRITE” commands, and the files were named NODES.DAT and ELEMS.DAT respectively according to the CRACK3D manual.

There are 3 input files for CRACK3D for simulations using the local re-meshing option. The first file, CRACK3D.MSH, contains the mesh information. The second file, CRACK3D.DAT, is used to define the analysis type, parameters for analysis, and boundary conditions. The last file used only for local re-meshing, CRACK3D.GEO, defines the boundary lines and surfaces in which the crack and re-meshing are contained.

Two CRACK3D.MSH and CRACK3D.GEO files were created. One set of files was for the initial mesh with 2 elements through the thickness at the crack front, and the second file was for the initial mesh with 4 elements through the thickness. MESH3D, a preprocessor for CRACK3D, was used to generate the CRACK3D.MSH file from the NODES.DAT and ELEMS.DAT files. CRACK3D.GEO was created with the help of ANSYS macros created by Dr. Weiming Lan. Output from the macros was compiled in Microsoft’s Notepad to create the CRACK3D.GEO file.

The CRACK3D.DAT file was created in Microsoft’s Notepad following the CRACK3D manual. A different CRACK3D.DAT file was created for each of the loading cases. The crack growth simulation was performed using local re-meshing for fatigue crack growth with VCCT and the MCS criterion for crack growth direction prediction. Parameters for  $\Delta K_{eq}$  defined in Eq 2.2 were chosen as follows:  $\gamma=0$ ,  $\gamma_1=1$ ,  $\gamma_2=1$ . Paris Law data from CRACK3D was not used for life prediction because the parameters for the

specific material and loading ratio being simulated were not currently available so that parameters for a different  $R$ -ratio were input. Also, the minimum element size, maximum element size, radius for the local re-meshing region, and increment for crack growth were defined. Boundary conditions were input for the specific loading case as described above in Section 3.2. As an example, the input files for the simulation for loading case  $\Phi = 45^\circ$  are contained in Appendix C.

The first simulations were performed to check for convergence of the crack path. For the  $15^\circ$  loading case, simulations were performed using the first initial mesh with 2 elements through the thickness and with the initial refined mesh containing 4 elements through the thickness. The minimum element sizes equivalent to  $1/2$  the thickness,  $1/4$  the thickness, and  $1/8$  the thickness were used, and crack growth increments for 0.001m and 0.002m were used. The maximum element size and re-meshing zone size remained constant at 0.006m and 0.012m respectively. Table 3.2 shows all the combinations of simulations performed to check convergence.

Table 3.2. All combinations of simulations performed to verify convergence of solution

Number of elements through thickness of initial mesh	Minimum element size [m]	Crack growth increment [m]
2	.003	0.001
		0.002
	.0015	0.001
		0.002
4	.0015	0.001
		0.002
	.00075	0.001
		0.002

Simulations were performed for loading cases  $\Phi = 15^\circ, 30^\circ, 45^\circ,$  and  $60^\circ$ . In some cases the local re-meshing operation in CRACK3D had numerical difficulties and could

not continue when the crack growth amount reached a specific value. In such cases, the local re-meshing parameters such as the minimum element size, the maximum element size, or re-meshing region size were adjusted slightly in a trial and error fashion. Also, sometimes the first initial mesh with 2 elements through the thickness was used while other times the refined mesh was used. After simulations were performed using CRACK3D, POST3D was run which converted results from CRACK3D into result files compatible with ANSYS for further post-processing.

### 3.4 POST PROCESSING OF SIMULATION RESULTS

CRACK3D.SIF is an output file from CRACK3D which contained  $x$ ,  $y$ ,  $z$ ,  $G_I$ ,  $G_{II}$ ,  $G_{III}$ ,  $K_I$ ,  $K_{II}$ , and  $K_{III}$  for each node along the crack front at each crack growth increment. CRACK3D.CXT contains the total reaction load for each crack growth increment. The total reaction load was summed over the nodes along the top line which the displacements were specified and was defined in the CRACK3D.DAT file. The data from these two files was imported to Microsoft Excel for post-processing of each simulation.

The reaction load from the prediction and the maximum load applied in the experiment were used to create a scale factor for the SIFs, since for nominally linear elastic conditions the load and SIF remain proportional. The SIF values were scaled to the experimental values for the maximum load. Then,  $\Delta K_I$  and  $\Delta K_{II}$  were calculated using Eq 1.4.

### 3.5 THEORETICAL RESULTS

The crack path for each of the simulations run for convergence check were plotted. No crack propagation occurred for the simulations with minimum element size of 1 /8 of the thickness (0.00075 m).

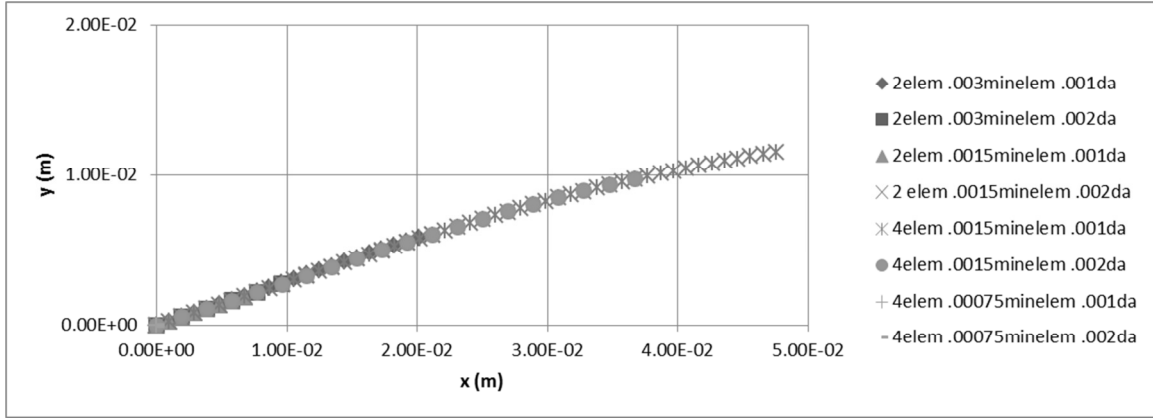


Figure 3.8. Crack path for  $\Phi = 15^\circ$  with various initial meshes, minimum element sizes, and crack increments.

Figure 3.9 shows the deformed mesh for loading case  $\Phi = 45^\circ$  for  $a = 0.03$  m with a close up of the crack and re-meshing zone.

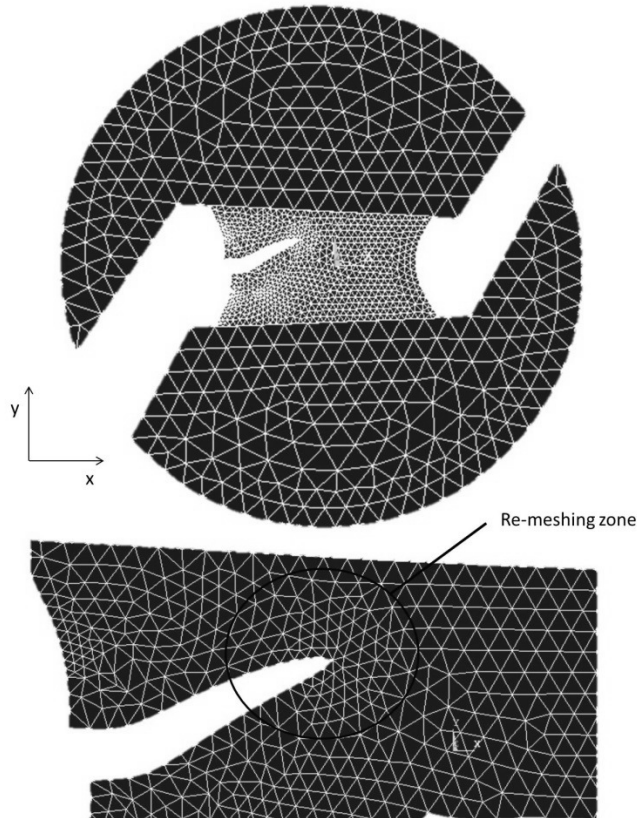


Figure 3.9. 2D view of the 3D deformed mesh for  $\Phi = 45^\circ$  (top) and a close up of the crack path and re-meshing zone around the crack front.

For loading cases  $\Phi = 15^\circ, 30^\circ, 45^\circ,$  and  $60^\circ$ , Figures 3.10-3.13 show the comparison between the experimental crack paths and the predicted crack paths.

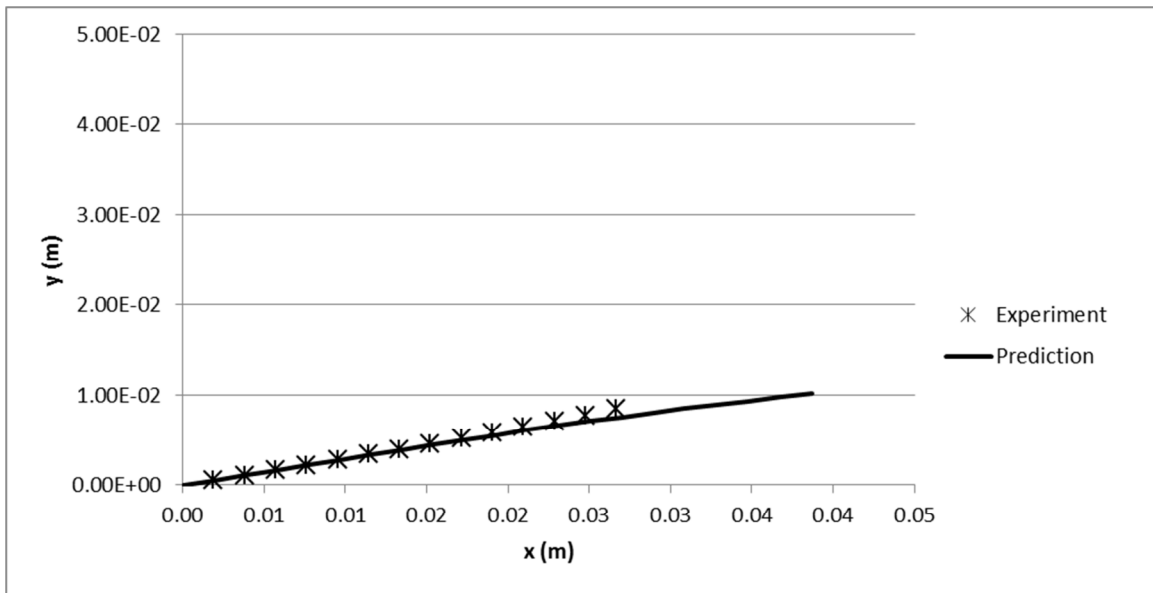


Figure 3.10. The experimental and predicted crack path for the  $15^\circ$  loading case.

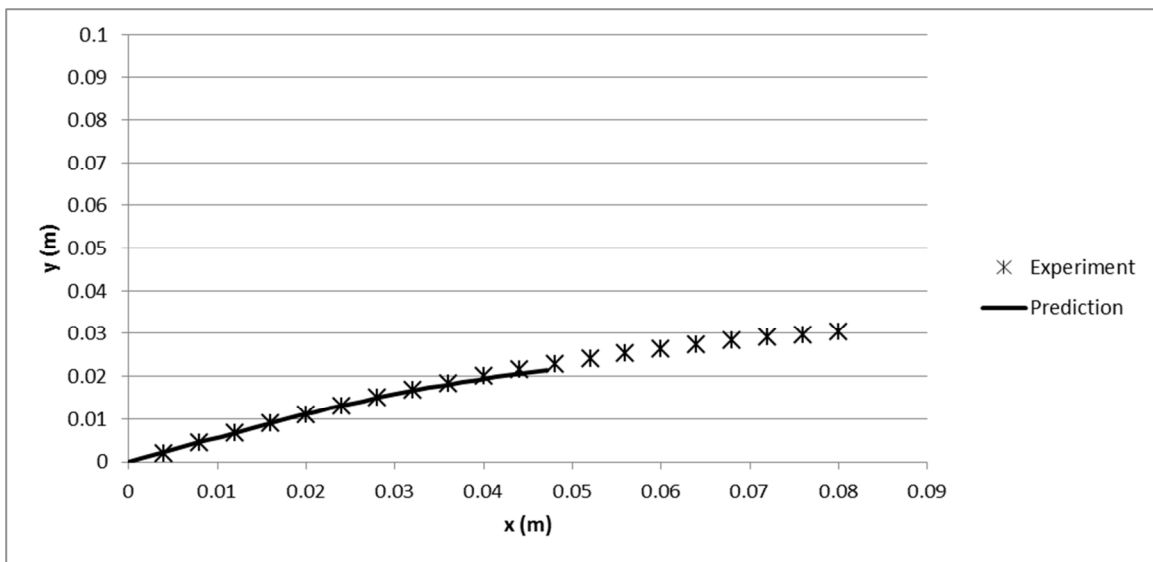


Figure 3.11. The experimental and predicted crack path for the  $30^\circ$  loading case.

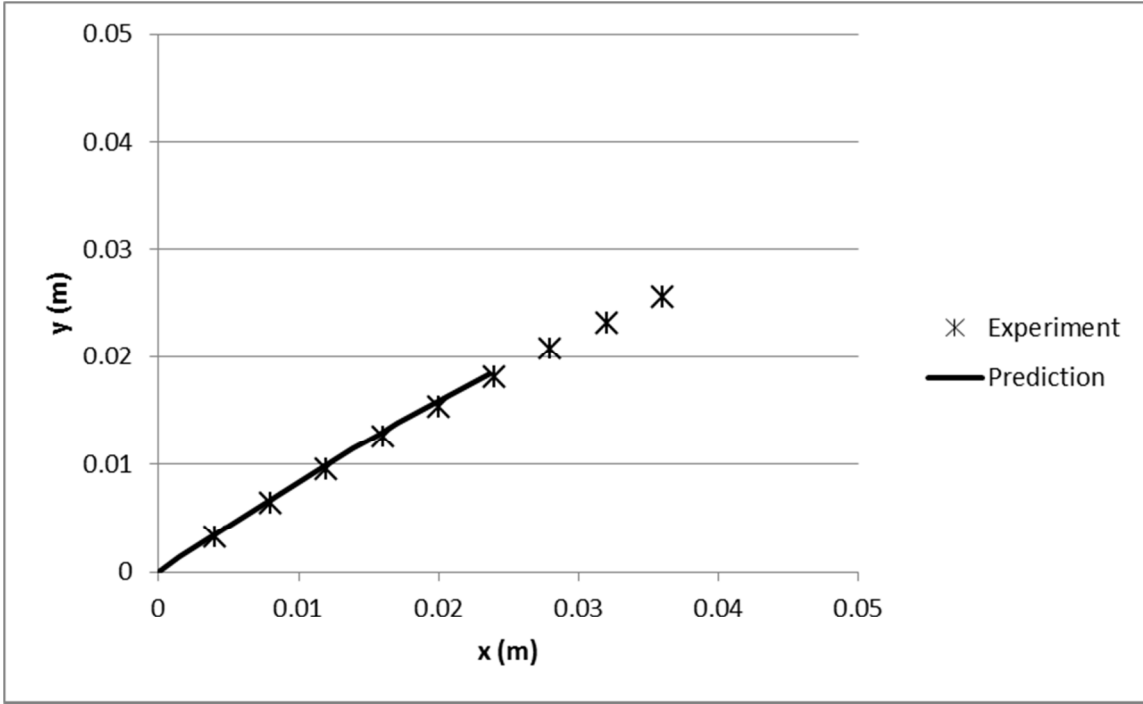


Figure 3.12. The experimental and predicted crack path for the 45° loading case.

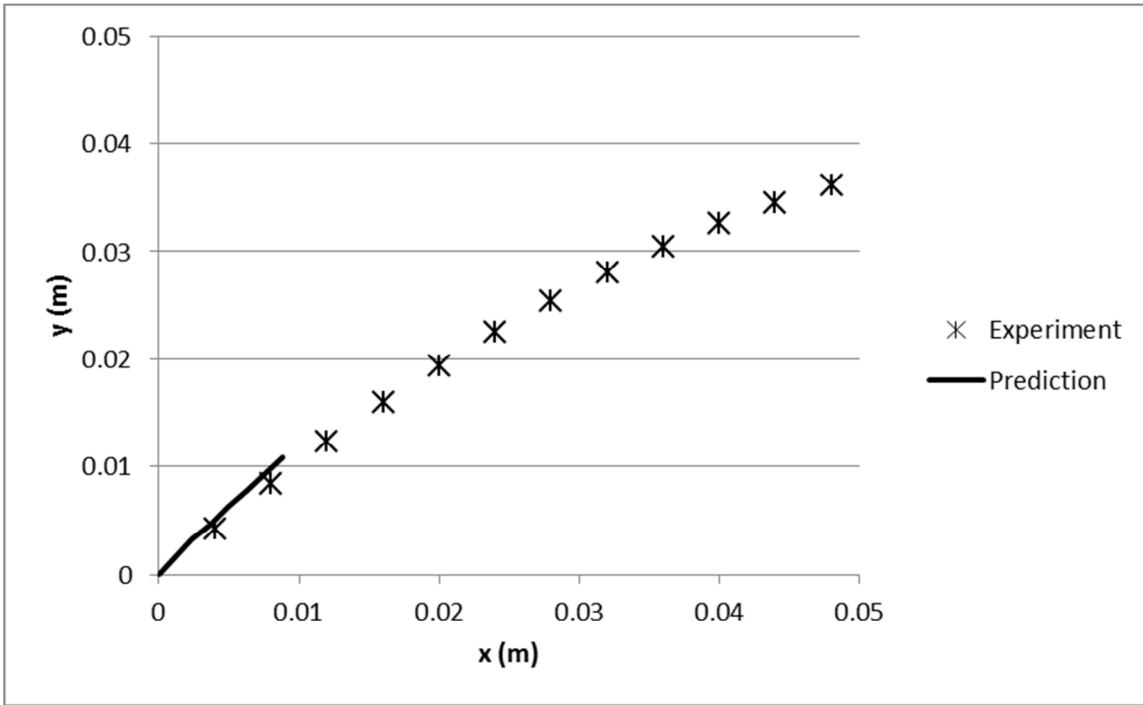


Figure 3.13. The experimental and predicted crack path for the 60° loading case.



The  $\Delta K_I$  and  $\Delta K_{II}$  for each loading cases  $\Phi = 15^\circ, 30^\circ, 45^\circ,$  and  $60^\circ$  are plotted along the crack length  $a$  in Figures 3.14-3.17.

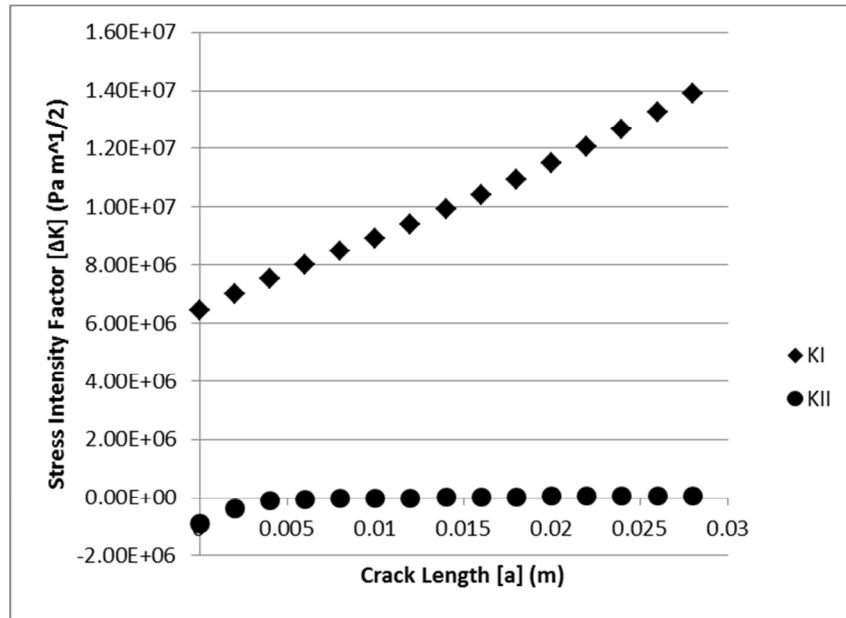


Figure 3.14. Plot of  $\Delta K_I$  and  $\Delta K_{II}$  along the crack path for the  $15^\circ$  loading case.

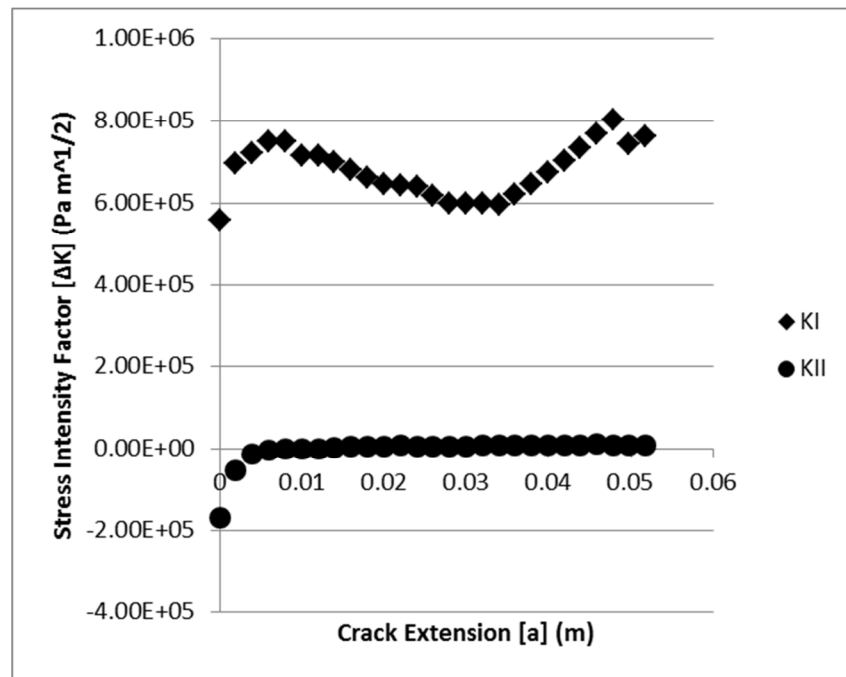


Figure 3.15. Plot of  $\Delta K_I$  and  $\Delta K_{II}$  along the crack path for the  $30^\circ$  loading case.

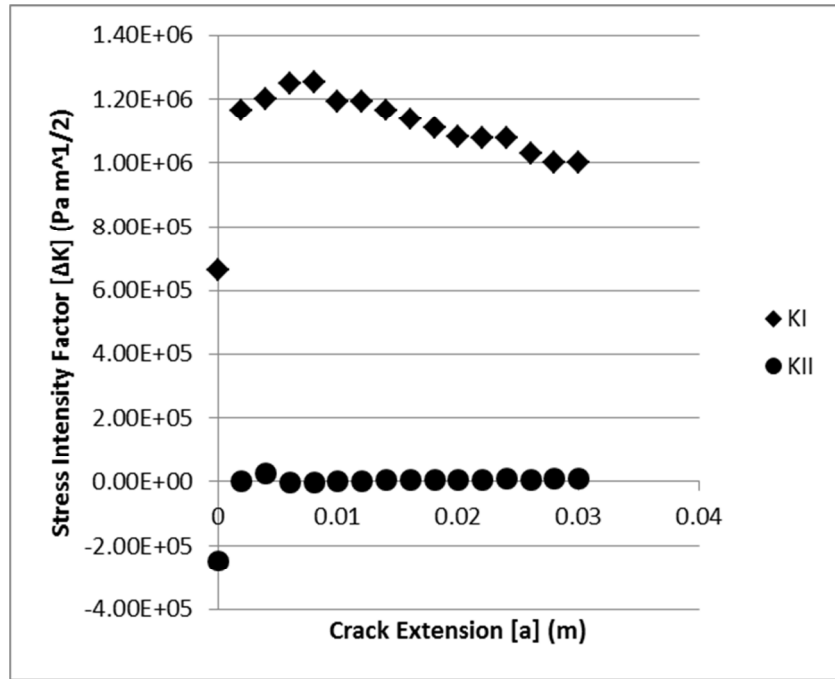


Figure 3.16. Plot of  $\Delta K_I$  and  $\Delta K_{II}$  along the crack path for the 45° loading case.

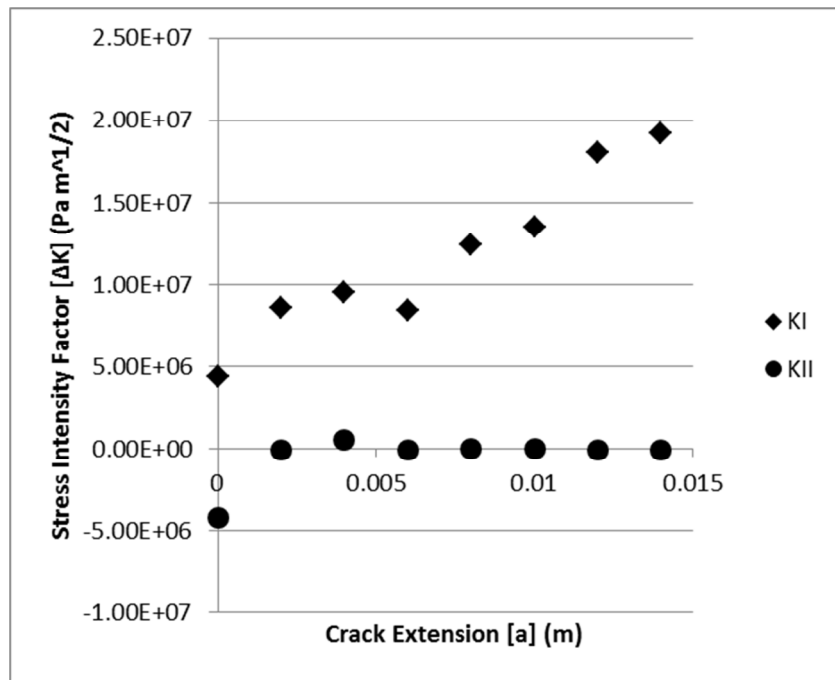


Figure 3.17. Plot of  $\Delta K_I$  and  $\Delta K_{II}$  along the crack path for the 60° loading case.

## CHAPTER 4

### DISCUSSION

#### 4.1 DISCUSSION OF EXPERIMENTAL RESULTS

There are several issues that require discussion. First, as noted in Section 2.5, the experimental crack paths were determined through an imaging-digitization-scaling process, where imaging was performed on the fatigue specimen after completion of the crack growth process. Results from this process gave consistent results for all loading angles,  $\Phi$ , and hence the resulting crack paths are used for direct comparison to the simulation predictions.

Secondly, regarding the crack growth process, as shown in Figures 3.10 to 3.13, there is excellent agreement between the measured and predicted crack growth paths. Furthermore, as shown in Figures 3.14 to 3.17, the simulation data shows that the crack growth process is occurring under nominally Mode I conditions, with  $\Delta K_{II} = 0$ , confirming that the fatigue crack tended to propagate under locally tensile conditions. For small loading angles, the crack growth direction is approximately perpendicular to the loading direction. However, as loading angle increases, the crack deviates from the perpendicular direction, implying that the local Mode I direction is no longer perpendicular to the loading angle. In fact, the curvilinear trend of the crack path which begins around  $x = 0.03\text{m}$  is the result of the influence of the loading process via the Arcan fixture on the stress field in the specimen. For  $\Phi = 30^\circ$ , Figure 4.1 shows a plot of

the crack path, with an additional line indicating the edge of the top fixture. Clearly, the Arcan fixture is sufficiently close to the crack path to have an influence on the local crack tip stress field in the specimen. In fact, the data for all loading angles show that as the crack approaches the steel fixture, it begins to turn and follow the edge of the upper fixture. Since the thickness of the Arcan fixture is much greater than the specimen and is manufactured from stainless steel, the crack turns to follow the “path of least resistance”. That is, it would require more energy to create new surfaces inside the fixture, so the crack turns to continue propagating in the aluminum specimen.

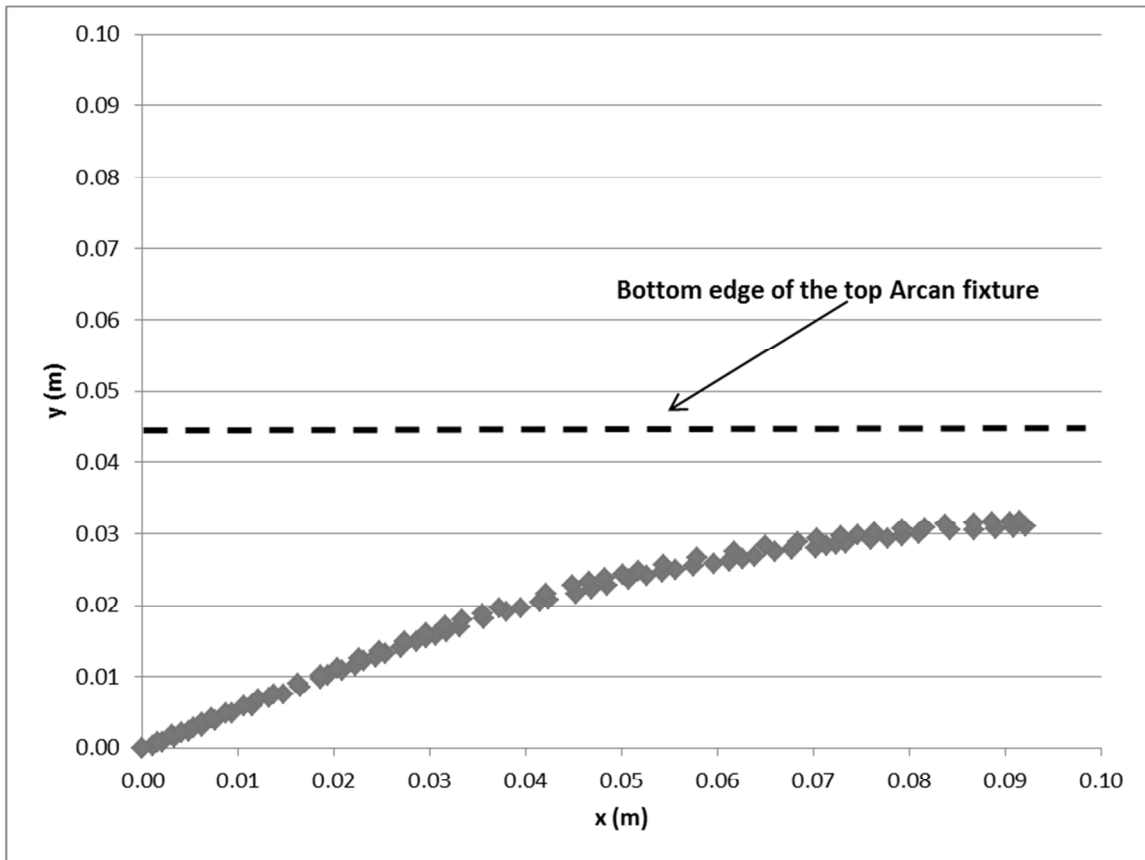


Figure 4.1. Plot of the crack path for  $\Phi = 30^\circ$  and edge of top fixture.

Thirdly, we were unable to observe crack growth for  $\Phi = 75^\circ$  and  $\Phi = 90^\circ$ . There are two plausible reasons why the crack did not grow in these cases. Firstly, other studies have shown that the stress distribution for the Arcan fixture is not uniform, with the largest gradients occurring in the  $75^\circ$  orientation [1]. The relatively short initial notch and initial pre-crack used in the experiment may have positioned the crack tip in an area of negative or low stress. This may have put the crack into compression or kept  $\Delta K_{eq}$  below  $\Delta K_{TH}$ , the threshold value required to initiate crack growth in the material. Preliminary FEA by the author was consistent with this observation. Another possibility for both  $\Phi = 75^\circ$  and  $90^\circ$  is that the Mode II component of the far field loading is larger than the Mode I component. In such cases, there may not be sufficient Mode I loading to open the crack tip and overcome friction and local plastic deformation along the contacting crack surfaces to allow the fatigue crack to propagate.

#### 4.2 DISCUSSION OF THEORETICAL RESULTS

Prior to discussing the results for the Arcan fatigue studies, it is important to note that benchmark studies have been performed, and it has been verified that CRACK3D is able to accurately predict the crack path for elastic plastic stable tearing using the Arcan fixture to achieve mixed-mode loading conditions using local re-meshing [2] [3] [4] [5] [6]. The direction of crack extension for stable tearing is predicted with a different criteria, crack opening displacement [2] [3] [4] [5] [6], while as discussed here, VCCT and MCS criterion are used in predicting the direction of fatigue crack propagation.

An important aspect of the simulations studies is to confirm consistency in the predicted crack path for each loading angle. The convergence of the predicted crack path

for  $\Phi = 15^\circ$  was checked, and the plot in Figure 3.8 shows there is good agreement in the predicted crack path for all combinations of initial meshes, crack growth increments, and minimum element sizes, with the lone exception of simulations where minimum element size was 0.00075m. It was observed that the various combinations did not provide the same amount of crack growth, though the results from all combinations gave generally consistent trends in crack growth path across all converged simulations.

It is worth noting that, for each simulation, at a certain point CRACK3D was unable to re-mesh the volume according to the re-meshing criteria provided in the code. Thus, the program was terminated at this point. Though the precise reason for the inability to continue propagating the fatigue crack is not fully understood, it must be stated that there is no theoretical reason why there should be an issue in the ability to predict the direction of crack propagation for longer amounts of crack extension using MCS criterion. Since other fatigue studies were performed with CRACK3D where the crack grew all the way across a different specimen geometry, the most likely reason for the limited amount of crack extension is that CRACK3D could not arrange elements in an acceptable manner inside the size of the re-meshing region for this specimen geometry to “match” the surrounding, un-meshed region. The effect of overall geometry is most clearly evident in Figure 4.1. Here, as the crack propagated, the re-meshing region approached the interface of the specimen and the fixture (see Fig 4.1). Since the code currently does not have the ability to define internal boundaries to control the re-meshing near material boundaries, as the re-meshing region approached the fixture boundary, the program could not re-mesh the region satisfactorily, resulting in termination of the crack growth process. Even with these issues, our convergence analysis does show that any of

these combinations of re-meshing parameters will give similar results when re-meshing is possible.

As shown in Figures 3.10 – 3.13, the predicted and experimental crack paths are in good agreement with each other in the region where re-meshing was achievable. As commented above regarding the robustness of CRACK3D, the simulations with the longest predicted crack path for each loading case were reported.

As noted previously, Figures 3.14 – 3.17 show that  $\Delta K_{II}$  quickly goes to zero along the predicted crack path, conditions that are consistent with the use of MCS criterion for predicting the direction of crack growth. For the first case,  $\Phi = 15^\circ$ , for the segment of crack path considered, the load was held constant, and so it is expected that  $\Delta K_I$  increases as shown in Fig. 3.14. However after crack slanting occurred for  $\Phi = 15^\circ$ ,  $\Delta K_I$  for the remaining experiments was held approximately constant (see Figure 3.15 – 3.17) via a modified method of load shedding to control the crack growth rate and maintain crack tip conditions that were nominally consistent with elasticity assumptions (e.g., small plastic zone relative to specimen dimensions). This was necessary so that crack growth occurred under conditions that reflected local stress intensity factor control. Figures 2.10 and 2.11 show that, for most crack lengths, the crack growth rate,  $da/dN$ , was maintained between the range of  $4 \times 10^{-5}$  mm/cycle and  $8 \times 10^{-5}$  mm/cycle, while also ensuring that the plastic zone size at the crack tip was small. For loading angle  $\Phi=45^\circ$ , near the end of the experiment the crack growth range was below these limits for  $da/dN$ , ranging from a minimum of  $2 \times 10^{-5}$  mm/cycle to a maximum of  $4 \times 10^{-5}$  (see Figures 2.11 and 2.12), which increased the time required to complete the experiment but did not alter the nominally elastic conditions required for these studies.

There are several comments to be made in regards to the efficiency and effectiveness of this modified method for controlling the change in SIF during the experiment. First, during one of the experiments the crack measurement system (microscope objectives, calipers) had to be re-zeroed. Since the crack lengths were re-measured and were slightly different (shorter) afterwards, one negative value for  $da/dN$  was obtained immediately after the re-zeroing process. This data point is not shown in Figure 2.10. Causes for other outliers in the data not shown in Figures 2.10 – 2.12 include (a) slight errors in the measured crack lengths while the experiment was being performed, (b) local variations in material (e.g., inclusions) or (c) other defects introduced in the manufacturing process. The effects identified in (a-c) were somewhat magnified in this study since crack growth was measured over very short cycle counts. Thus, changes in crack growth for any of the reasons noted above will appear as steep gradients in the  $\Delta a/\Delta N$  data. If crack growth had been averaged in over a much larger time frame (more cycles), then these effects would be more muted.

Secondly, the goal of this modified load shedding technique was to maintain an average constant  $da/dN$ , and so far it has been discussed that in general that goal was achieved with only a few outliers in the data. However for short segments of crack growth increasing or decreasing trends in  $da/dN$  exist (Figures 2.10- 2.12). These trends in the data can be seen in Figure 2.10 between 40 and 60 mm of crack length and in Figure 2.11 and 2.12 at the beginning of the experiment around 10 mm of crack length. However for the experiment with  $\Phi=60^\circ$ , in an effort to maintain the specified range of crack growth rates, the loads were increased too much and the crack growth rate jumped beyond the set limits. In this case, over short cycle counts the load was decreased until



$da/dN$  was back in the appropriate range. It is noted that these trends are the result of the difficulty in the practical application of this modified load shedding approach. The inability to precisely control  $da/dN$  means that  $\Delta K_{eq}$  does not actually remain constant during the experiment and could potentially result in  $\Delta K_{eq}$  becoming too large and the nominally elastic conditions may not be present.

Thirdly, this modified method is based on the premise that, according to Paris' Law (Eq 1.5),  $da/dN$  and  $\Delta K_{eq}$  are proportional. Thus, trends in the measured crack growth rate and the computed  $\Delta K_{eq}$  at the crack tip should be consistent throughout the crack growth process. To determine whether this approach gave consistent results, predicted values for  $K_I$  and  $K_{II}$  were scaled to the experimental values using the resultant load from the simulations and the maximum load applied during the experiment.  $\Delta K_I$  and  $\Delta K_{II}$  were then calculated using Eq 1.4 and  $\Delta K_{eq}$  was calculated according to Eq. 2.2 with parameters  $\gamma = 0$ ,  $\gamma_I = 1$ , and  $\gamma_2=1$  as discussed in Section 3.3. Figures 4.2-4.4 show a direct comparison of the scaled  $\Delta K_{eq}$  and the discrete crack growth rate recorded during the experiment,  $\Delta a/\Delta N$ , as a function of crack length,  $a$ .

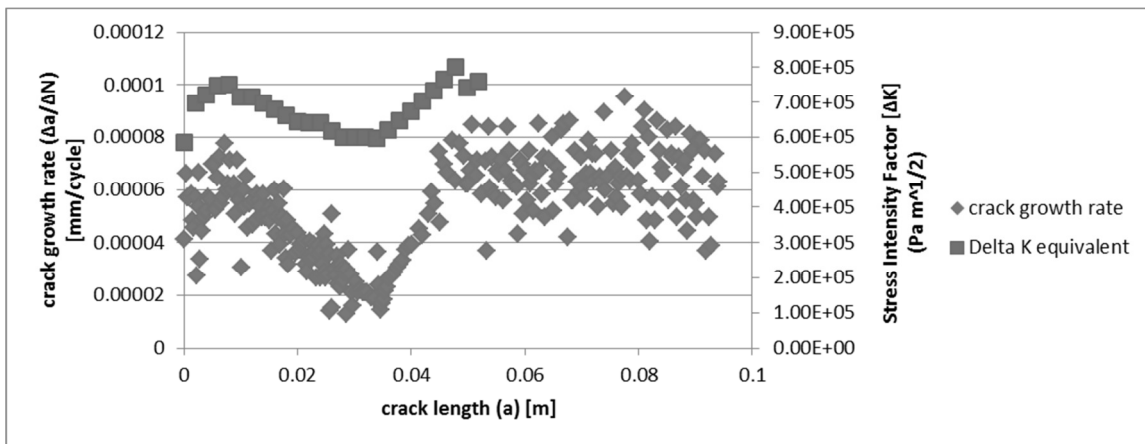


Figure 4.2. Plot of  $\Delta K_{eq}$  and  $da/dN$  along crack length for 30° loading case.

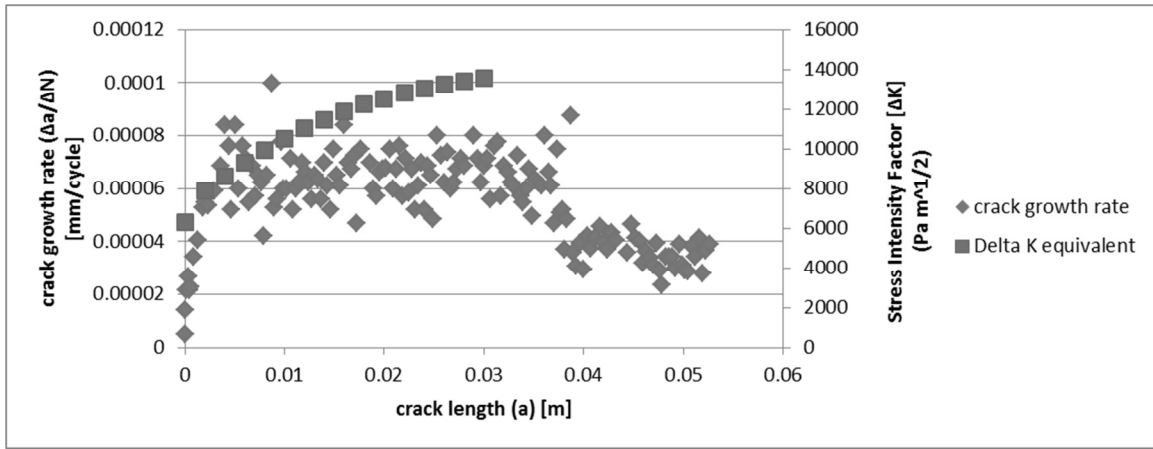


Figure 4.3. Plot of  $\Delta K_{eq}$  and  $da/dN$  along crack length for  $45^\circ$  loading case.

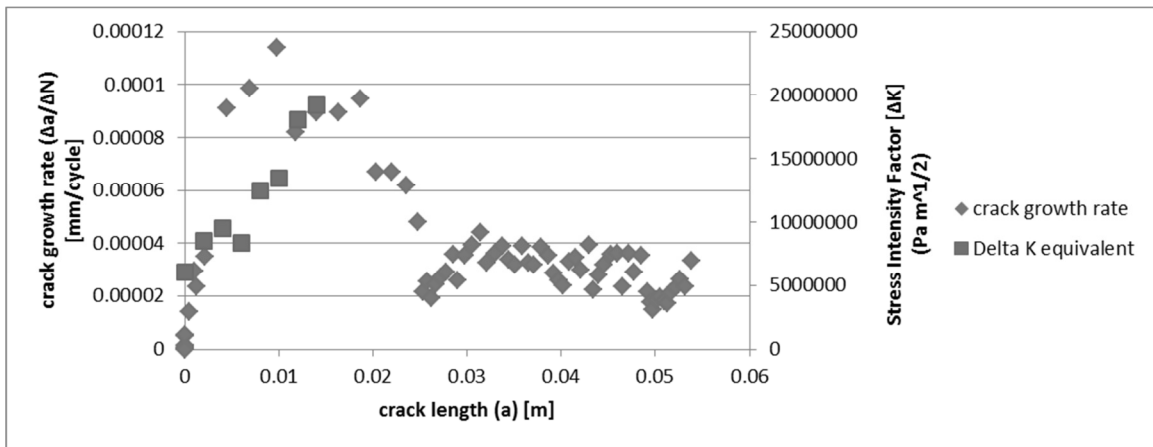


Figure 4.4. Plot of  $\Delta K_{eq}$  and  $da/dN$  along crack length for  $60^\circ$  loading case.

Figures 4.2 - 4.4 show that the trend in  $\Delta K_{eq}$  follows the trend in  $da/dN$  as expected from Paris' Law (Eq 1.5).

Fourthly, load control was used to determine the crack growth rate since it directly alters  $\Delta K_{eq}$  (instead of using crude empirical expressions). Even though Figs 4.2-4.4 show that  $\Delta K_{eq}$  is a viable approach, there are some challenges associated with controlling the crack growth rate in experiments. First, some previous knowledge of loading and crack growth rates must be known to have a starting point to ensure that the experiments are

within the nominally linear elastic range. Secondly, the procedure is quite time consuming since the crack must be measured approximately every 5,000 cycles until the rate is established (see the first 20mm of crack growth for  $\Phi = 45^\circ$  and  $\Phi = 60^\circ$  in Figures 4.3 and 4.4 respectively). After the rate was established, it was determined that the load had to be decreased  $\approx$  every 25,000 cycles. Thus, unless an automated approach is developed to perform the crack growth rate estimations, the experimentalist must be near the test stand through the duration of the experiment to take the crack growth measurements for these cycle count intervals. Thirdly, the loads should be adjusted in small increments to minimize changes that might cause crack closure or other unwanted effects. This slow adjustment can take 10 to 20 mm of crack growth, and thus the  $\Delta K_{eq}$  is not held constant for the whole experiment. In this regard, it is noted that a more accurate way of predicting loads for the experiment is to use  $\Delta K_{eq}$  values obtained by performing FEA before the experiment to determine the maximum loads which can be applied and the maximum crack length the loads can be used before the plastic zone at the crack tip becomes too large. However, this would require a priori knowledge of the crack path, which is generally not the case. Fourthly, though simply changing loads slightly to maintain modest crack growth rates is effective for the experimentalist, changing the loads during the experiment also makes the simulations more time consuming. Multiple models and simulations have to be performed for each load step if load boundary conditions are used. This avoids more post-processing on the back end. However, it is very time consuming to build multiple models. A more efficient way is to use one simulation in displacement control. The reaction load can be scaled at each crack growth increment with the loads for the corresponding crack growth increment from the

experiment. This requires more post- processing and accurate records for the experiment. Also, if the  $R$ -ratio is not held constant through the experiment, the life prediction using Paris' Law (Eq. 1.5) can become more difficult since the constants  $C$  and  $m$  may depend on  $R$  and the rate data must be known for the whole range of loading ratios used.

Finally, it is noted that standards for fatigue testing suggest that they be run using constant amplitude (constant load levels) [31]. This method is most efficient experimentally and in simulations. However with  $\Delta K$  increasing, not as much crack growth can be obtained before large plasticity occurs at the crack tip or stable tearing initiates. As discussed earlier, the method of load shedding selected in this study was chosen to obtain the maximum amount of crack extension under nominally elastic conditions.

## CHAPTER 5

### CONCLUSIONS

Fatigue crack growth experiments have been performed successfully on an edge-cracked Arcan specimen manufactured from 2024-T351 aluminum and subjected to far-field mixed mode I/II loading with loading angles  $\Phi = 15^\circ, 30^\circ, 45^\circ, \text{ and } 60^\circ$ . Experimental results from these experiments included (a) crack paths, (b) crack extension vs. fatigue cycles and (c) the minimum and maximum loads applied during each cycle. Results show that (a) the two degree of freedom slide apparatus developed especially for these experiments is an effective and efficient method of determining experimental crack paths, eliminating the cumbersome post-processing requirements for digitizing and scaling images of the specimen after crack extension had occurred, (b) crack extension occurs along different curvilinear paths for each loading angle, extending from the original crack tip towards the upper Arcan grip, (c) initial kinking angle of the fatigue crack indicates that the local Mode I direction deviates the direction perpendicular to the loading angle as the Mode II component of loading increases, (d) the load shedding process used to maintain crack growth rates in a specific range that was used for  $\Phi = 30^\circ, 45^\circ, \text{ and } 60^\circ$  is consistent with controlling  $\Delta K_{eq}$ , as shown through direct comparison of experimental crack growth rates and predicted  $\Delta K_{eq}$  values at points along the measured crack paths, and (e) further study is required for loading angles  $\Phi = 75^\circ$  and  $90^\circ$  where fatigue crack growth was not observed experimentally.

Simulations of the fatigue crack growth process for the Arcan fixture-specimen combination have been performed using a custom-finite element code for fracture analysis, CRACK3D. In this code, VCCT is used to quantify the local stress intensity factors and the MCSC is used to determine the direction of current crack extension. Results from the simulations show that (a) CRACK3D is an effective simulation platform for fatigue crack growth in many cases, (b) the re-meshing algorithms in CRACK3D are not readily adaptable for crack growth near material junctures where there are significant differences in element size; the ability to handle such cases is currently being developed, but not yet available, (c) direct comparison of the experimental results and predictions indicate that the measured and CRACK3D predicted crack paths using local re-meshing to maintain accuracy in the local fields are in excellent agreement over the range of crack growth where the simulations were convergent, (d) predictions using an idealized notch and pre-crack yield little error between the predicted and experimental crack paths, and(e) indicate that the direction of crack propagation corresponds to the direction which maximizes Mode I and minimized Mode II, which is consistent with results from previous studies [1] [2]

## CHAPTER 6

### RECOMMENDATIONS FOR FUTURE WORK

It is recommended that for the current study, a life prediction be completed using the predicted values of  $\Delta K_{eq}$  and Paris' Law (Eq 1.5) to compare predicted  $da/dN$  to the experimental crack growth rate ( $\Delta a/\Delta N$ ). Currently the rate data for  $R = 0.4$  is not available, and the Paris' Law constants for  $R = 0.4$  are unknown for Al-2024-T351. The data may be available through the last version of AFGROW [1] available to the public [2], though this has not yet been verified.

While the current study only compared the predicted crack path to the experimental crack path, it is recommended that further experiments be conducted to obtain the experimental values of  $\Delta K_I$  and  $\Delta K_{II}$  for comparison. It has been shown that DIC is an accurate method of obtaining SIFs around the crack tip [3] [4]. Currently, a script in MATLAB [5] has been created to use the displacement field around the crack tip, accounting for rigid body translation and rotation, in William's solution [6] to iteratively solve for values of  $K_I$  and  $K_{II}$  until the solution has reached convergence using a Levenberg-Marquardt least squares [7] to solve the over-determined system of non-linear equations. To implement this experimentally, it is suggested that for various crack lengths, the specimen be statically loaded to zero, maximum, and minimum force with images of the crack tip being obtained at each load level. Then DIC can be conducted for

the maximum and minimum loads with the image at zero force being the reference image. From those displacement fields corresponding to the maximum and minimum loads and the method of determining the SIFs discussed above,  $\Delta K_I$  and  $\Delta K_{II}$  can then be determined using Eq 1.2.

It has been suggested that further experiments be conducted to experimentally determine SIFs, and as discussed in the Chapter 4, the current method of load prediction is not efficient while performing the experiment or for the simulation post-processing. Since the crack paths have now been determined, it is recommended that for this second set of experiments FEA be used to predict the loads for the experiment. It would be optimal to keep constant amplitude for as long a crack path as possible before having to perform load shedding to keep the number of load shedding steps at a minimum.

The current work did not provide sufficient information for  $\Phi = 75^\circ$  and  $90^\circ$ , and further work is necessary to understand fatigue crack propagation when Mode II is dominant. It is recommended that a finite element model of the fixture and specimen with the fatigue pre-crack should be built. The stress fields at the crack tip should be evaluated. To perform experiments at these loading angles, it may be necessary to grow the fatigue pre-crack further, into a region of higher stress or to keep the pre-crack in the same location but apply higher loads.

If previous studies show that fatigue cracks always propagate in Mode I, it may be because the local Mode I direction is the only direction providing enough opening without friction of the surfaces caused by the shear and cyclic loading. Stable tearing cracks propagate in the Mode II direction for  $\Phi = 75^\circ$  and  $90^\circ$ , but for that case, there is



no cyclic loading and the loads are increased to overcome the friction between the surfaces. It is recommended that a tensile bar should be used to apply a constant  $K_I$  to the Arcan specimen while cyclic loading is applied at either  $\Phi = 75^\circ$  and  $90^\circ$ , as shown in Figure 6.1. That tension may provide enough opening of the crack tip to allow the fatigue crack to propagate in the Mode II direction . The tension bar should be manufactured carefully such that it does not interfere with the ability to visibly track the crack growth. It should also not apply a moment to the specimen if possible, and the tensile load applied should be able to be held constant and be able to be adjusted. The load should also be recorded, and it is suggested that a strain gage to be used to do so.

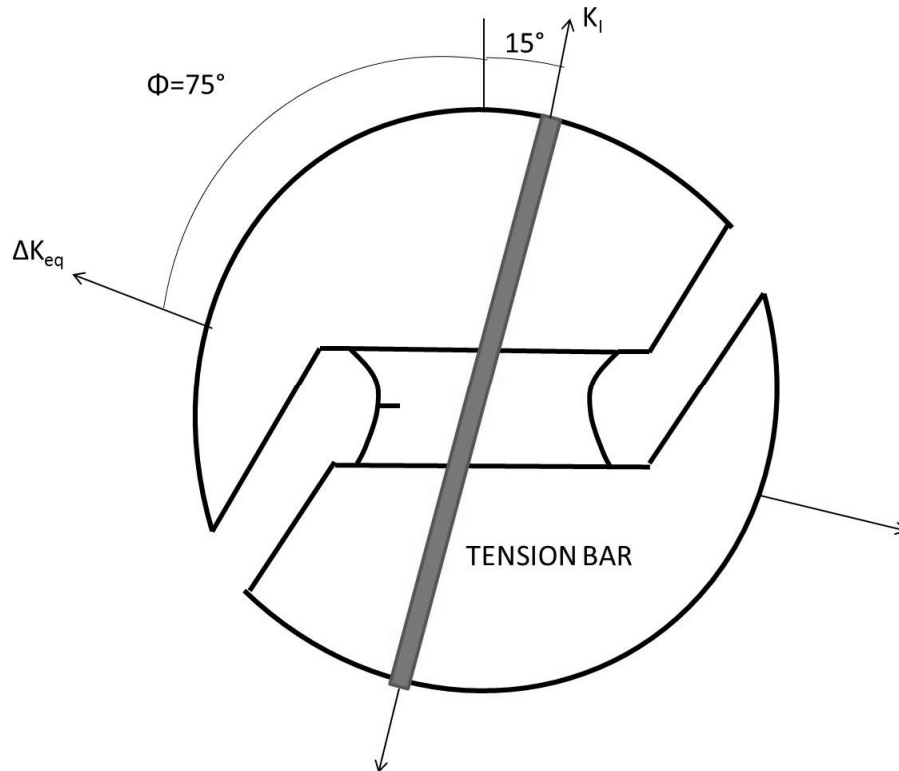


Figure 6.1. Schematic of Arcan fixture with proposed tension bar for  $\Phi = 75^\circ$ .

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## APPENDIX A – EXPERIMENTAL DATA RECORDED

Table A.1. Experimental data recorded data for  $\Phi = 15^\circ$ .

Cycles(N)	X-position of crack tip [m]	Maximum Load (Pmax) [N]	Minimum Load (Pmin) [N]
Pre-cracking			
50,000	0.0067	48308	7962
100,000	0.0073		
130,000	0.0081	45630	7558
150,000	0.0092	43392	7166
170,000	0.0100	38277	6592
177,000	0.0105	36867	6183
187,000	0.0110		
197,000	0.0115	35532	5907
202,000	0.0116	34153	5725
212,000	0.0121		
217,000	0.0123	32650	5542
221,000	0.0124		
225,400	0.0126		
225,400	0.0126		
Marking Crack Front			
231,000	0.0127	28384	14203
240,000	0.0127		
260,000	0.0128		
275,000	0.0129		
290,000	0.0129		
Test			
290,000	0.0124		
300,000	0.0127	31756	5396
315,000	0.0132		
330,000	0.0137		
345,000	0.0143		
360,000	0.0149		
375,000	0.0157		
390,000	0.0165		

405,000	0.0173		
420,000	0.0183		
430,000	0.0190		
440,000	0.0198		
450,000	0.0206		
460,000	0.0215		
470,000	0.0225		
480,000	0.0236		
487,500	0.0243		
495,000	0.0254		
500,000	0.0260		
505,000	0.0267		
510,000	0.0274		
515,000	0.0282		
520,000	0.0290		
525,000	0.0299		
530,000	0.0308		
535,000	0.0318		
540,000	0.0329		
545,000	0.0340		
550,000	0.0351		
555,000	0.0364		
560,000	0.0378		
565,000	0.0394		
570,000	0.0413		
573,000	0.0423		
576,000	0.0434		
579,000	0.0448		
582,000	0.0461		
585,000	0.0477		
588,000	0.0490		
591,000	0.0507		
594,000	0.0526		
597,000	0.0551		
598,000	0.0560		
598,300	0.0561	29109	4568
598,600	0.0562		
598,800	0.0563		
599,000	0.0566		
599,300	0.0567	27116	4528
599,600	0.0568		

599,900	0.0568		
600,200	0.0570		
600,400	0.0571		
600,600	0.0571		
600,800	0.0573		
601,000	0.0573	25261	4551
601,400	0.0574		
601,800	0.0575		
602,200	0.0577		
602,600	0.0578		
603,000	0.0578	23335	4551
603,600	0.0581		
604,200	0.0582		
604,800	0.0583		
605,400	0.0585		
606,000	0.0586	20982	4551
606,600	0.0586		
607,600	0.0588		
608,600	0.0590		
608,800	0.0590		
609,600	0.0591	19172	4546
610,800	0.0593		
612,000	0.0594		
613,800	0.0596		
615,600	0.0602		
617,400	0.0604	17433	4519
618,200	0.0604	15760	4515
620,600	0.0605		
623,000	0.0607		
623,400	0.0609		
630,000	0.0618	14234	4453
638,000	0.0620	12677	4453
640,000	0.0621		
662,000	0.0626	11468	4462
676,000	0.0628		
696,000	0.0630	9791	4444
726,000	0.0630		
761,000	0.0632		
781,000	0.0633		
801,000	0.0634		
821,000	0.0634	7580	6076



861,000	0.0634		
925,000	0.0634		
1,005,000	0.0634	8447	6761
1,030,000	0.0634	8447	5916
1,045,000	0.0634	19990	16005
1,057,000	0.0636	.	
1,067,000	0.0637		
1,081,000	0.0638		
1,093,000	0.0639	8332	4728
1,123,000	0.0640		
1,195,000	0.0640		
1,270,000	0.0640		
1,320,000	0.0640	9070	4453

Table A.2. Experimental data recorded data for  $\Phi = 30^\circ$ .

Cycles(N)	X- position of crack tip [m]	Maximum Load (Pmax) [N]	Minimum Load (Pmin) [N]
Pre-cracking			
50,000	0.0060	45012	4897
100,000	0.0063		
150,000	0.0095		
160,000	0.0098	36386	4813
170,000	0.0103		
175,000	0.0105	35279	4622
180,000	0.0107		
185,000	0.0109	34100	4599
190,000	0.0111		
197,000	0.0114		
202,000	0.0116	32948	4613
210,000	0.0119		
215,000	0.0122		
220,000	0.0124	31662	4546
225,000	0.0127		
Marking Crack Front			
240,000	0.0128	30768	15391
250,000	0.0129		
285,000	0.0130		
315,000	0.0131		
340,000	0.0133		
Testing			
345,000	0.0113	29945	4791
346,000	0.0115	51155	45416
348,000	0.0117		
350,000	0.0118	49108	19648
355,000	0.0121	47334	18949
360,000	0.0124		
365,000	0.0127		
370,000	0.0130	45461	18304
375,000	0.0132		
380,000	0.0134	43899	17691
385,000	0.0137		
387,000	0.0139		
390,000	0.0140	42765	17183

395,000	0.0142		
397,000	0.0144		
400,000	0.0145	41453	16610
405,000	0.0147		
410,000	0.0149		
416,000	0.0152		
422,000	0.0156		
428,000	0.0159		
434,000	0.0162		
440,000	0.0166		
445,000	0.0170		
450,000	0.0173	40087	16085
455,000	0.0176		
460,000	0.0179	38886	15578
465,000	0.0182		
470,000	0.0185		
475,000	0.0189	37561	15071
480,000	0.0192		
485,000	0.0195	36493	16859
490,000	0.0199		
495,000	0.0202	35501	14221
500,000	0.0205		
505,000	0.0207	34536	13852
510,000	0.0211		
515,000	0.0214		
520,000	0.0217	33642	13460
525,000	0.0218		
530,000	0.0221		
535,000	0.0224	32752	13118
540,000	0.0227		
545,000	0.0229	31876	12784
550,000	0.0232		
555,000	0.0236		
560,000	0.0238	31071	12464
565,000	0.0241		
570,000	0.0243	30301	12144
575,000	0.0246		
580,000	0.0249		
585,000	0.0252	29545	11850

590,000	0.0254		
595,000	0.0257		
600,000	0.0260	28824	11561
605,000	0.0262		
610,000	0.0265	28104	11298
615,000	0.0267		
620,000	0.0270		
625,000	0.0272	27437	11032
630,000	0.0274		
635,000	0.0277		
640,000	0.0279	26778	10760
645,000	0.0282		
650,000	0.0285		
655,000	0.0287	26133	10533
660,000	0.0289		
665,000	0.0291		
667,000	0.0293	25515	10315
672,000	0.0295		
677,000	0.0296		
682,000	0.0299		
685,000	0.0300		
695,000	0.0304	24923	10080
705,000	0.0309		
715,000	0.0312	24341	9835
725,000	0.0317		
735,000	0.0320	23789	9626
740,000	0.0322		
750,000	0.0326	23229	9399
755,000	0.0328		
765,000	0.0332	22708	9199
770,000	0.0333		
775,000	0.0335		
780,000	0.0337		
785,000	0.0339	22210	8981
795,000	0.0342		
802,500	0.0345		
807,500	0.0347	21712	8790
812,500	0.0348		
817,500	0.0350		

827,500	0.0354		
832,500	0.0355	21205	8585
837,500	0.0357		
842,500	0.0359		
847,500	0.0360	20782	8363
852,500	0.0362		
857,500	0.0364		
862,500	0.0365		
867,500	0.0367		
872,500	0.0369	20346	8162
877,500	0.0370		
882,500	0.0372		
887,500	0.0374		
892,500	0.0374	19924	7962
897,500	0.0377		
902,500	0.0378		
908,500	0.0369		
914,500	0.0382	19439	7802
920,500	0.0383		
926,500	0.0385		
932,500	0.0387		
938,500	0.0389	19021	7651
944,500	0.0391		
950,500	0.0392		
956,500	0.0394		
962,500	0.0396	18598	7464
968,500	0.0397		
974,500	0.0399		
980,500	0.0401		
986,500	0.0402		
995,500	0.0404	18207	7313
1,005,500	0.0407		
1,015,500	0.0410		
1,025,500	0.0412	17811	7144
1,035,500	0.0414		
1,055,500	0.0419		
1,075,500	0.0423	17415	6984
1,095,500	0.0427		
1,115,500	0.0432	17050	6828

1,135,500	0.0436	16676	6681
1,155,500	0.0440		
1,185,500	0.0446	16307	6543
1,215,500	0.0452		
1,230,500	0.0475		
	0.0000		
1,245,500	0.0457		
1,260,000	0.0460		
1,275,000	0.0462		
1,290,000	0.0465		
1,305,000	0.0468		
1,320,000	0.0471		
1,335,000	0.0474		
1,350,000	0.0478		
1,365,000	0.0483		
1,380,000	0.0487		
1,395,000	0.0491		
1,410,000	0.0496		
1,425,000	0.0501		
1,440,000	0.0507		
1,455,000	0.0513		
1,470,000	0.0519		
1,495,000	0.0530		
1,510,000	0.0536		
1,530,000	0.0546		
1,540,000	0.0552		
1,550,000	0.0558		
1,560,000	0.0565		
1,564,000	0.0567		
1,574,000	0.0574		
1,584,000	0.0581		
1,594,000	0.0589		
1,604,000	0.0595		
1,614,000	0.0603		
1,624,000	0.0610	14973	5978
1,629,000	0.0613	14572	5907
1,634,000	0.0616		
1,639,000	0.0620		
1,644,000	0.0624		

1,654,000	0.0630	14265	5769
1,659,000	0.0634		
1,669,000	0.0640	13959	5627
1,679,000	0.0647		
1,684,000	0.0649	13727	5480
1,689,000	0.0653		
1,694,000	0.0656	13425	5342
1,699,000	0.0660		
1,704,000	0.0663	13118	5236
1,709,000	0.0665		
1,714,000	0.0669		
1,719,000	0.0672		
1,724,000	0.0676		
1,729,000	0.0678	12838	5120
1,734,000	0.0682		
1,739,000	0.0686		
1,744,000	0.0690	12544	5004
1,749,000	0.0693	12277	4915
1,754,000	0.0696		
1,759,000	0.0699	12010	4804
1,764,000	0.0702		
1,769,000	0.0704		
1,774,000	0.0708		
1,779,000	0.0711		
1,784,000	0.0714	11752	4706
1,789,000	0.0717		
1,794,000	0.0720		
1,799,000	0.0724		
1,804,000	0.0727	11499	4599
1,809,000	0.0730		
1,814,000	0.0733		
1,819,000	0.0736		
1,824,000	0.0740		
1,829,000	0.0744	11223	4506
1,834,000	0.0747	10983	4390
1,839,000	0.0750		
1,844,000	0.0753	10787	4297
1,854,000	0.0760		
1,859,000	0.0764		

1,864,000	0.0767	10556	4186
1,869,000	0.0770		
1,874,000	0.0773		
1,879,000	0.0776		
1,884,000	0.0780		
1,889,000	0.0785	10289	4110
1,894,000	0.0790	10066	4026
1,899,000	0.0792	9622	3852
1,904,000	0.0796		
1,909,000	0.0799	9408	3763
1,914,000	0.0802		
1,919,000	0.0806		
1,924,000	0.0809	9194	3679
1,929,000	0.0812	8994	3603
1,934,000	0.0815		
1,939,000	0.0819	8785	3510
1,944,000	0.0821		
1,949,000	0.0825		
1,954,000	0.0829		
1,959,000	0.0832	8585	3438
1,964,000	0.0836		
1,969,000	0.0839	8385	3363
1,974,000	0.0842		
1,979,000	0.0845	8198	3292
1,984,000	0.0848		
1,989,000	0.0852		
1,994,000	0.0856	8016	3216
1,999,000	0.0859		
2,004,000	0.0862		
2,009,000	0.0866		
2,014,000	0.0870	7642	3003
2,019,000	0.0872	7464	2936
2,024,000	0.0876		
2,029,000	0.0879	7295	2874
2,034,000	0.0882		
2,039,000	0.0885		
2,044,000	0.0888		
2,049,000	0.0893		
2,054,000	0.0896	7122	2874



2,059,000	0.0899		
2,064,000	0.0903	6966	2811
2,069,000	0.0907		
2,074,000	0.0910	6806	2749
2,079,000	0.0914	6637	2682
2,084,000	0.0917		
2,089,000	0.0920	6486	2620
2,094,000	0.0924		
2,099,000	0.0928	6352	2549
2,104,000	0.0931	6205	2487
2,109,000	0.0935		
2,114,000	0.0937	6058	2433
2,119,000	0.0940		
2,124,000	0.0943		
2,129,000	0.0945		
2,134,000	0.0949		
2,139,000	0.0953	5907	2220
2,144,000	0.0957		
2,149,000	0.0960	5765	2313
2,154,000	0.0964		
2,159,000	0.0968	5765	2313
2,164,000	0.0971	5636	2260
2,169,000	0.0974		
2,174,000	0.0978	5494	2060
2,179,000	0.0982	5356	1993
2,184,000	0.0985	5231	2100
2,189,000	0.0988		
2,194,000	0.0991	5098	2046
2,199,000	0.0995		
2,204,000	0.0998	4982	1828
2,209,000	0.1001		
2,214,000	0.1003	4849	1953
2,219,000	0.1007		
2,224,000	0.1011	4728	1899
2,229,000	0.1014	4613	1855
2,234,000	0.1017		
2,239,000	0.1019		
2,244,000	0.1023	4515	1646
2,249,000	0.1027	4390	1761

2,254,000	0.1030	4284	1721
2,259,000	0.1034	4177	1486
2,264,000	0.1036		
2,274,000	0.1041		
2,284,000	0.1045		
2,294,000	0.1052		
2,299,000	0.1055	4075	1624
2,304,000	0.1058		

Table A.3. Experimental data recorded data for  $\Phi = 45^\circ$ .

Cycles(N)	X-Position of Crack Tip [m]	Maximum Load (Pmax) [N]	Minimum Load (Pmin) [N]
Pre-cracking			
25,000	0.0064	68356	26961
40,000	0.0073		
55,000	0.0082	61025	24278
70,000	0.0092	57871	23353
80,000	0.0097	55469	22157
90,000	0.0103	53005	21280
100,000	0.0108	51528	20440
110,000	0.0113		
120,000	0.0116	47124	18981
130,000	0.0121		
140,000	0.0125	45372	18264
Marking Crack Front			
145,000	0.0094	43851	26338
150,000	0.0094	47787	28722
160,000	0.0095		
170,000	0.0096		
185,000	0.0098		
Testing			
195,000	0.0094	42458	17081
200,000	0.0094	47089	18273
205,000	0.0095		
210,000	0.0096	48930	19661
215,000	0.0098		
220,000	0.0099		
225,000	0.0100		
235,000	0.0103		
245,000	0.0107		
255,000	0.0113		
260,000	0.0115		
265,000	0.0118		
275,000	0.0124		
285,000	0.0131		
290,000	0.0135		
295,000	0.0139	47823	18949
300,000	0.0141	46511	18313

305,000	0.0146		
310,000	0.0149	44927	17691
315,000	0.0152		
320,000	0.0156	43014	17148
325,000	0.0159	41497	16605
330,000	0.0162		
335,000	0.0165		
340,000	0.0168	40163	16107
345,000	0.0171		
350,000	0.0173		
355,000	0.0177	38700	15622
360,000	0.0182		
365,000	0.0184	37441	15168
370,000	0.0187		
375,000	0.0191		
380,000	0.0194	36391	14750
385,000	0.0197		
390,000	0.0200		
395,000	0.0203	35363	14337
400,000	0.0206		
405,000	0.0209		
410,000	0.0213		
415,000	0.0216	34429	13950
420,000	0.0219		
425,000	0.0222		
430,000	0.0225	33504	13598
435,000	0.0228		
440,000	0.0231		
445,000	0.0235		
450,000	0.0238	32708	13269
455,000	0.0240		
460,000	0.0244		
465,000	0.0247	31778	12882
470,000	0.0250		
475,000	0.0254		
480,000	0.0258	30986	12557
485,000	0.0261		
490,000	0.0265		
495,000	0.0267	30190	12242

500,000	0.0271		
505,000	0.0270	29461	11943
510,000	0.0277		
515,000	0.0280		
520,000	0.0283	28740	11650
525,000	0.0286		
530,000	0.0289		
535,000	0.0293		
540,000	0.0296		
545,000	0.0300		
550,000	0.0303	28042	11370
555,000	0.0306		
560,000	0.0310		
565,000	0.0313	27361	11107
570,000	0.0316		
575,000	0.0319	26694	10823
580,000	0.0323		
585,000	0.0325		
590,000	0.0328		
595,000	0.0332		
600,000	0.0335	26089	10578
605,000	0.0338		
610,000	0.0341		
615,000	0.0344		
620,000	0.0348		
625,000	0.0351		
630,000	0.0354	25484	10324
635,000	0.0358		
640,000	0.0361	24892	10080
645,000	0.0364		
650,000	0.0367		
655,000	0.0371		
660,000	0.0374	24318	9857
665,000	0.0437		
670,000	0.0380		
675,000	0.0384		
680,000	0.0388	23856	9626
685,000	0.0391		
690,000	0.0395		

695,000	0.0398		
700,000	0.0401	23220	9399
705,000	0.0405		
710,000	0.0409		
715,000	0.0411	22708	9190
720,000	0.0415		
725,000	0.0418		
730,000	0.0421	22192	8990
735,000	0.0424		
740,000	0.0428		
745,000	0.0431	21205	6361
750,000	0.0434		
755,000	0.0437		
760,000	0.0440		
765,000	0.0442		
770,000	0.0446		
775,000	0.0449		
780,000	0.0452		
785,000	0.0456		
790,000	0.0459	20742	8394
795,000	0.0462	20275	8207
800,000	0.0464		
805,000	0.0468		
810,000	0.0471	19505	7847
815,000	0.0473	18972	7678
820,000	0.0475	18554	7504
825,000	0.0478		
830,000	0.0482		
835,000	0.0482	18162	7344
840,000	0.0484	17762	7188
850,000	0.0487	17379	7037
860,000	0.0491		
870,000	0.0494		
880,000	0.0498		
890,000	0.0502		
900,000	0.0506		
910,000	0.0511		
920,000	0.0514	16997	6877
930,000	0.0518		

940,000	0.0522		
950,000	0.0527	16997	6739
960,000	0.0601		
970,000	0.0534	16267	7473
980,000	0.0538		
990,000	0.0542	15929	6445
1,000,000	0.0547		
1,010,000	0.0551	15920	6285
1,020,000	0.0554	15244	6170
1,030,000	0.0557		
1,040,000	0.0561		
1,050,000	0.0564	14910	6023
1,060,000	0.0568		
1,070,000	0.0571	14586	6170
1,080,000	0.0573		
1,090,000	0.0577		
1,100,000	0.0580	14265	5769
1,110,000	0.0583		
1,120,000	0.0586		
1,130,000	0.0590		
1,140,000	0.0593	13959	5640
1,150,000	0.0596		
1,160,000	0.0599		
1,170,000	0.0603		
1,180,000	0.0606		
1,190,000	0.0610		
1,200,000	0.0613	13660	5525
1,210,000	0.0617		
1,220,000	0.0621		

Table A.4. Experimental data recorded data for  $\Phi = 60^\circ$ .

Cycles(N)	X-Position of Crack Tip [m]	Y-Position of Crack Tip [m]	Maximum Load (Pmax) [N]	Minimum Load (Pmin) [N]
Pre-cracking				
30,000	0.0072	0.0006	67346	26769
45,000	0.0081	0.0006		
60,000	0.0090	0.0006	60451	24350
75,000	0.0097	0.0006	55798	23282
90,000	0.0104	0.0005		
105,000	0.0109	0.0005	50292	21182
120,000	0.0114	0.0006		
135,000	0.0119	0.0007	46177	19688
150,000	0.0124	0.0007	46270	19221
165,000	0.0129	0.0007		
Marking Crack Front				
175,000	0.0131	0.0006	54646	33984
185,000	0.0133	0.0007		
240,000	0.0133	0.0007	42859	11908
Testing				
240,000	0.0070	0.0113		
250,000	0.0071	0.0113	43370	18789
260,000	0.0071	0.0113	47565	19763
270,000	0.0071	0.0114	51292	21182
280,000	0.0071	0.0114		
290,000	0.0071	0.0114	53739	20377
300,000	0.0071	0.0116	61817	20156
330,000	0.0076	0.0115		
350,000	0.0081	0.0115		
360,000	0.0084	0.0116		
385,000	0.0093	0.0120		
410,000	0.0115	0.0123		
435,000	0.0140	0.0129	55149	22188
460,000	0.0168	0.0135	50999	20426
485,000	0.0189	0.0141		
510,000	0.0211	0.0148	41609	16605
535,000	0.0234	0.0155	39415	15625
560,000	0.0257	0.0162	36617	14759
585,000	0.0274	0.0167	32630	13225



610,000	0.0291	0.0173	30977	12562
635,000	0.0306	0.0179	29469	11943
660,000	0.0318	0.0183	26716	10814
685,000	0.0324	0.0186	23224	9399
705,000	0.0329	0.0187		
725,000	0.0333	0.0189		
745,000	0.0338	0.0191		
765,000	0.0343	0.0193		
785,000	0.0349	0.0195		
805,000	0.0356	0.0198		
825,000	0.0361	0.0200		
845,000	0.0368	0.0203		
865,000	0.0376	0.0206		
885,000	0.0385	0.0210		
905,000	0.0391	0.0212	22192	8990
930,000	0.0400	0.0217		
950,000	0.0408	0.0220		
970,000	0.0415	0.0223	21205	8585
990,000	0.0421	0.0227		
1,010,000	0.0429	0.0230		
1,030,000	0.0436	0.0234	20271	8661
1,050,000	0.0442	0.0237		
1,070,000	0.0450	0.0241		
1,090,000	0.0457	0.0243	19403	7856
1,110,000	0.0462	0.0247		
1,130,000	0.0468	0.0249		
1,150,000	0.0472	0.0252	18585	7486
1,170,000	0.0479	0.0255		
1,190,000	0.0486	0.0259		
1,210,000	0.0492	0.0263	17771	7188
1,230,000	0.0500	0.0268		
1,250,000	0.0504	0.0270	17001	7313
1,270,000	0.0510	0.0274		
1,290,000	0.0516	0.0278		
1,310,000	0.0523	0.0282		
1,330,000	0.0531	0.0283	16276	6868
1,350,000	0.0535	0.0291		
1,370,000	0.0543	0.0296		
1,390,000	0.0548	0.0299	15578	6592

1,410,000	0.0555	0.0303		
1,440,000	0.0562	0.0308	14906	5863
1,460,000	0.0565	0.0311	14270	5760
1,480,000	0.0568	0.0313		
1,500,000	0.0572	0.0315		
1,520,000	0.0576	0.0318		
1,540,000	0.0580	0.0321		
1,560,000	0.0583	0.0324		
1,580,000	0.0587	0.0327		
1,600,000	0.0592	0.0330		
1,620,000	0.0597	0.0334		
1,640,000	0.0602	0.0339		
1,660,000	0.0609	0.0343		

Table A.5. Experimental data recorded data for  $\Phi = 90^\circ$ .

Cycles(N)	X-Position of Crack Tip [m]	Y-Position of Crack Tip [m]	Maximum Load (Pmax) [N]	Minimum Load (Pmin) [N]
Pre-cracking				
0	0.0065	0.0000	67128	26961
5,000	0.0065	0.0000		
15,000	0.0066	0.0002		
25,000	0.0071	0.0003		
35,000	0.0078	0.0002		
45,000	0.0082	0.0001	60207	24341
55,000	0.0088	0.0001		
65,000	0.0092	0.0001	54855	22148
75,000	0.0095	0.0000		
85,000	0.0101	0.0000		
95,000	0.0104	0.0001	50470	20453
105,000	0.0109	0.0001		
120,000	0.0116	0.0001	46849	18972
130,000	0.0110	0.0001		
140,000	0.0124	0.0001		
150,000	0.0128	0.0001		
Testing				
150,000	0.00024	0.01259	44206.4	17677.2
155,000	0.00024	0.01262		
165,000	0.00024	0.01262		
175,000	0.00024	0.01262		
185,000	0.00024	0.01273		
195,000	0.00024	0.01273		
205,000	0.00024	0.01273		
225,000	0.00024	0.01273		
235,000	0.00024	0.01273		

## APPENDIX B –DETAILS AND TIPS FOR CRACK3D INPUT FILES GENERATION

First, the geometry has to be created to accommodate the crack and the three zones in the mesh, and steps for building the geometry will now be discussed. The volume for the top fixture was created by creating keypoints at the position where the end of each of the two diagonal lines and the straight line segment for the bottom edge of the fixture are located. Then keypoints were created at a radius of 0.14 m and in approximately  $7^\circ$  intervals being sure that keypoints were positioned at the same location where the center of the pin holes are on the fixture. A spline through the keypoints was used to create the arc of the top edge of the fixture. The front area was created by selecting the lines. The volume was created by extruding the area along the normal direction in an amount equivalent to the thickness of the grip. To create the bottom fixture, the top fixture volume was then reflected about the x-axis then again about the y-axis. The volume, areas, lines, and keypoints remaining from the first reflection was deleted. Material properties for the fixture were assigned to the two volumes.

The initial geometry was created using multiple volumes in order to create a crack and volumes for the three zones of the mesh: a locally structured finer mesh immediately around the crack front to facilitate 3D VCCT, a far-field coarser mesh away from the crack front region, and a graded mesh in a transitional zone between the far-field coarser mesh and the locally structured finer mesh, as shown in Figure B.1. Specifically, volumes 1, 2, 3, and 4 are for the locally structured mesh, volumes 5, 6, 7, and 8 are for the transitional zone with a graded mesh, and volumes 9, 10, 11, and 12 are for the far-field

mesh. The crack is created in ANSYS by creating the bottom surfaces (seen as lines in the 2D view in Fig. B.1) of volumes 5 and 1 in the same position as the top surfaces (seen as lines in Fig. B.1) of volumes 8 and 4. The lines do not share the same end point at the edge of the specimen however they share the same keypoints at the crack front (i.e. the coincident vertices and edge of volumes 1, 2, 3, and 4 share the same keypoints and line).

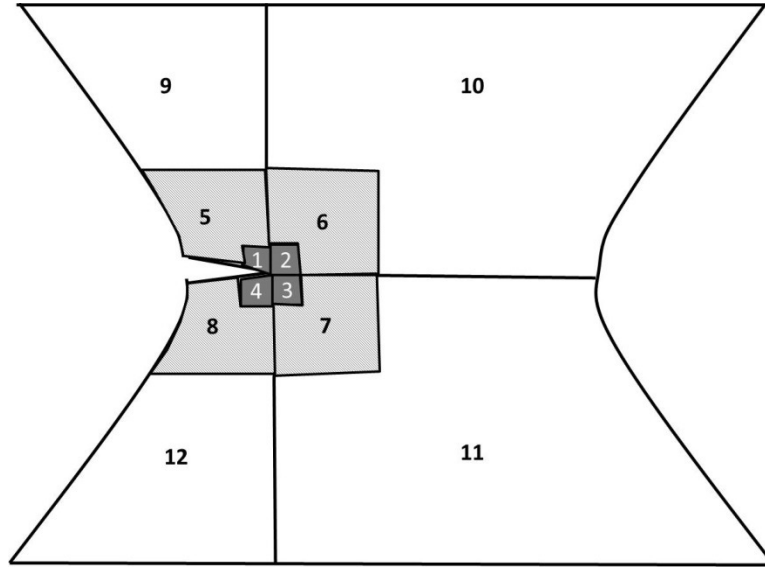


Figure B.1. A 2D schematic diagram of volumes in the specimen region used to create the initial finite element mesh.

The volumes were created in the same way the fixture volumes were created: by creating keypoints, then lines, areas, and extruding the areas along the normal to create volumes. Material properties were assigned to all the volumes of the specimen geometry. To create connectivity, all the volumes for the fixture and specimen were glued together with the exceptions of volumes attached to the areas which are the top and bottom crack surfaces: volume 5 was not glued to 8 and volume 1 was not glued to 4.

Secondly, A structured mesh was created in volumes 1, 2, 3, and 4 by first meshing the surface areas with mapped triangular element. Size division for the elements was set

such that each edge of the volume was one element length so that the minimum element size is one half of the thickness. The volume was then freely meshed using 10 noded tetrahedral elements, and the area elements were deleted. The element and node numbers were compressed. Then the size division for each of the remaining lines for the remaining volumes of the specimen was set to be maximum element size. The remaining volumes for the specimen and fixture were freely meshed using 10 noded tetrahedral elements.

Thirdly, The CRACK3D.GEO file was created. The CRACK3D.GEO file defines the surface boundary of the geometry by listing nodes attached to areas and lines on the outside of the geometry. That is, areas or lines, contained within the geometry do not need to be included. In defining surface areas for CRACK3D, surface areas may be comprised of multiple areas in ANSYS. In defining surface lines, lines may also be comprised of multiple lines, however, lines can not border two surface areas. Take for example, in Figure B.1, all areas 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 are considered one area, and boundary lines do not include the lines such as the line separating areas 4 from 6. It is also only necessary to include the boundaries in which the crack propagation is contained. Since the crack will not grow into the fixture, only specimen boundaries were included in the .GEO file. In listing the lines, the nodes must be listed along boundary lines in order along the line, and in listing the elements on the boundary surfaces, this file requires that area normals in ANSYS be pointing away from the geometry listing the nodes attached to the elements on the boundary surfaces in a counter clockwise manner. The area normals were plotted, and it was verified that for each surface area of the specimen and fixture, the area normal was pointing outwards. If a normal was not pointing outwards, the area normal was reversed.

Fourthly, for the re-meshing parameters in the CRACK3D.DAT file, based on benchmark studies, it is sufficient to have a minimum structured element size of one half of the specimen thickness. It is suggested that the local re-meshing zone be 6 to 10 times larger than the minimum element size. The maximum element size may be coarse but sufficient for transferring the load data to the crack front and equivalent to the element size in the far field mesh. It is also suggested that the far field mesh be uniform so that during crack extension the maximum element size in the re-meshing zone is constant and consistent with far field mesh elements. In CRACK3D the amount of crack extension (crack increment) at a crack growth step is chosen by the user as an input.

## APPENDIX C –FILES ASSOCIATED WITH CRACK3D FOR $\Phi = 45^\circ$

This appendix contains the input and output files for CRACK3D for the  $45^\circ$  loading case simulation. However CRACK3D.MSH is not included since that file was generated via ANSYS and MESH3D software while CRACK.GEO and CRACK.DAT are included since those files are user generated according to the CRACK3D user manual. The only output files reported where those used for post-processing: CRACK3D.SIF and CRACK3D.CXT.

### CRACK3D.DAT

Arcan 4 elem through thickness

3 1 0 2 1 -100  
1 1 2 2 1 1 10

1 361

1 1001  
2 1001  
1 7.11e10 0.33  
2 2.07e11 0.30

2 6 45 1

0 1 1  
0.5 1.8131e-24 -1 0 0 5.3366 0 1

5 2 -1 3 0 0.0520 -1 -1

0.0015 0.006 0.012 0.002

1	16649	16664	2
2	16780	16759	2
3	16781	16760	2
4	16782	16761	2
5	16783	16762	2
6	16784	16763	2
7	16785	16764	2
8	16786	16765	2



9	16612	16597	2
10	16650	16665	2
11	16770	16749	2
12	16787	16766	2
13	16772	16751	2
14	16788	16767	2
15	16774	16753	2
16	16789	16768	2
17	16776	16755	2
18	16611	16596	2
19	16647	16647	-1
20	16705	16705	-1
21	16704	16704	-1
22	16703	16703	-1
23	16702	16702	-1
24	16701	16701	-1
25	16700	16700	-1
26	16699	16699	-1
27	16595	16595	-1
28	16678	16678	-2
29	16791	16791	-2
30	16808	16808	-2
31	16793	16793	-2
32	16809	16809	-2
33	16795	16795	-2
34	16810	16810	-2
35	16797	16797	-2
36	16626	16626	-2
37	16679	16679	-2
38	16801	16801	-2
39	16802	16802	-2
40	16803	16803	-2
41	16804	16804	-2
42	16805	16805	-2
43	16806	16806	-2
44	16807	16807	-2
45	16627	16627	-2

2 -1 -1

Loading for 45 deg case

1 1 1 10 0 0 0

12196 110 -0.0000000707 0.0000000707 0  
13255 110 -0.0000000707 0.0000000707 0  
13254 110 -0.0000000707 0.0000000707 0  
13253 110 -0.0000000707 0.0000000707 0  
11546 110 -0.0000000707 0.0000000707 0  
1233 110 0 0 0  
2499 110 0 0 0  
2498 111 0 0 0  
2497 110 0 0 0  
583 110 0 0 0

5

1233 2499 2498 2497 583

100 0.5 0 0 0

## CRACK3D.GEO

9 21 0 6

76 78 96 50 52 58 58 1078 1080

1

141 30 139 345 344 140  
34 14 347 35 334 333  
34 135 137 340 136 341  
30 32 139 31 343 344  
346 141 126 335 142 329  
346 26 28 338 27 336  
34 137 32 341 342 33  
347 303 133 332 304 330  
347 133 135 330 134 331  
137 139 32 138 343 342  
28 30 141 29 345 339  
347 135 34 331 340 333  
347 14 303 334 302 332  
327 346 126 337 329 328  
26 346 327 338 337 326  
28 141 346 339 335 336  
7697 5864 7699 7645 7636 7637  
6721 5896 5898 7665 5897 7664  
5857 5864 7697 5863 7645 7647  
5912 7698 6705 7643 7641 7649  
5880 5882 6737 5881 7680 7681  
6713 5904 5906 7657 5905 7656  
5876 6743 5874 7686 7687 5875  
14 303 7698 302 7642 7644  
6735 5882 5884 7679 5883 7678  
6715 5902 5904 7659 5903 7658  
5902 6715 6717 7659 6716 7660  
5886 6733 5884 7676 7677 5885  
6733 6735 5884 6734 7678 7677  
6737 6739 5880 6738 7682 7681  
5872 6745 6747 7689 6746 7690  
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6719 5900 6717 7662 7661 6718  
7699 6698 7565 7640 7566 7638  
6711 5906 5908 7655 5907 7654  
7699 6753 6698 7639 6754 7640  
7699 5864 6753 7636 7648 7639  
7565 5857 7697 7564 7647 7646  
7565 7697 7699 7646 7637 7638  
5878 6739 6741 7683 6740 7684  
6745 5874 6743 7688 7687 6744  
14 7698 5912 7644 7643 5913  
133 7698 303 7635 7642 304  
5888 6729 6731 7673 6730 7674  
6707 5910 5912 7651 5911 7650  
6707 5912 6705 7650 7649 6706  
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6715 5904 6713 7658 7657 6714

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5866 6751 6753 7695 6752 7696  
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6727 5892 6725 7670 7669 6726  
5870 6749 5868 7692 7693 5869  
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5870 6747 6749 7691 6748 7692  
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6751 5866 5868 7695 5867 7694  
6729 5890 6727 7672 7671 6728  
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6707 6709 5910 6708 7652 7651

2

1990 2269 2292 2270 2278 2280  
2013 2292 2291 2272 2277 2281  
2269 2291 2292 2282 2277 2278  
1993 2292 2013 2279 2272 2285  
2005 2013 2291 2014 2281 2283  
2013 2011 1993 2012 2286 2285  
1995 2011 1997 2288 2287 1996  
2293 2007 2001 2273 2006 2271  
1993 1990 2292 1992 2280 2279  
2293 2003 1991 2275 2004 2276  
2005 2291 2269 2283 2282 2268  
2007 1999 2009 2284 2289 2008  
2009 1999 1997 2289 1998 2290  
2011 2009 1997 2010 2290 2287  
2007 2293 1999 2273 2274 2284  
2001 2003 2293 2002 2275 2271  
1993 2011 1995 2286 2288 1994  
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2112 2027 2110 2347 2346 2111  
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2017 2358 2356 2295 2296 2304  
2019 2120 2017 2355 2307 2018  
2057 2080 2082 2316 2081 2317  
2053 2086 2051 2321 2322 2052

2067 2358 2120 2299 2298 2121  
2061 2063 2076 2062 2311 2312  
2017 2120 2358 2307 2298 2295  
2065 2357 2072 2302 2300 2308  
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2055 2082 2084 2318 2083 2319  
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2114 2023 2025 2350 2024 2349  
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2031 2033 2106 2032 2341 2342  
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2358 2067 2069 2299 2070 2297  
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2015 2017 2356 2016 2304 2306  
2106 2108 2031 2107 2343 2342  
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2106 2033 2104 2341 2340 2105  
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2063 2065 2074 2064 2309 2310  
2086 2053 2084 2321 2320 2085  
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2357 2005 2269 2303 2268 2301  
2005 2357 2065 2303 2302 2066  
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7581 5845 5861 7573 5862 7571  
6702 6686 7581 6703 7567 7570  
7581 6686 7562 7567 7563 7572  
7580 6700 5859 7574 7577 7569

4

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5416 5559 5608 5560 5598 5595

5302 5608 5559 5599 5598 5558  
2001 2003 5607 2002 5601 5603  
5304 5298 5590 5303 5578 5576  
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5589 5420 5588 5584 5587 5579  
5543 5412 5592 5544 5572 5570  
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5308 5306 5588 5307 5582 5583  
5588 5306 5589 5582 5581 5579  
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5308 5588 5591 5583 5564 5574  
5302 5591 5559 5575 5573 5558  
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17960 18138 17962 18135 18137 17974  
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18113 18139 18138 18130 18129 18134  
18113 18036 18139 18114 18132 18130  
18138 17960 18113 18135 18112 18134

5

280 8 6 275 7 274  
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4 1 282 3 270 268  
160 162 281 161 276 277  
280 164 285 278 258 253  
162 164 280 163 278 279  
282 1 248 270 247 269  
282 281 4 255 272 268  
284 160 281 263 277 254  
284 127 160 264 159 263  
248 127 284 249 264 262  
283 8 280 266 275 256



281 280 6 271 274 273  
164 155 285 165 260 258  
283 2 8 267 9 266  
283 280 285 256 253 257  
281 6 4 273 5 272  
281 162 280 276 279 271  
248 284 282 262 261 269  
282 284 281 261 254 255  
155 251 285 252 259 260  
285 251 283 259 265 257  
365 1 39 356 40 354  
39 131 365 361 348 354  
366 131 127 352 132 353  
37 26 364 36 360 359  
126 129 364 128 357 349  
248 1 365 247 356 355  
366 127 248 353 249 351  
364 327 126 358 328 349  
364 26 327 360 326 358  
129 37 364 362 359 357  
365 131 366 348 352 350  
366 248 365 351 355 350  
39 37 131 38 363 361  
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10223 10222 6654 10020 10087 10084  
10187 10186 2112 10057 10159 10156  
2092 10197 10196 10136 10047 10139  
10193 10194 10246 10050 9982 9981  
10247 10194 10195 9979 10049 9980  
10248 10247 10195 9894 9980 9977  
10181 9609 10182 10171 10168 10062  
10237 10183 10184 10002 10009 9999  
10244 10243 10191 9898 9988 9985  
10218 10219 10269 10024 9934 9933  
10188 2108 10189 10155 10152 10055  
10250 10198 10251 9974 9971 9891  
10338 2072 1990 9645 2071 9646  
10258 10205 10206 9957 10038 9958  
6668 6666 10216 6667 10099 10098  
10202 10201 2082 10042 10129 10126  
10227 10226 6646 10016 10079 10076  
10267 10268 10311 9874 9799 9798  
10255 10300 10299 9820 9757 9823  
10329 10311 10312 9682 9744 9683  
10285 10286 10323 9771 9728 9727  
10324 10286 10287 9725 9770 9726  
18690 18702 18763 18703 18722 18711  
143 18765 157 18717 18716 158  
18764 18762 18763 18712 18725 18718  
18755 18700 18698 18749 18699 18750  
18692 18760 18694 18731 18739 18693

18761 18758 18759 18735 18740 18736  
18758 18756 18759 18742 18741 18740  
18759 16356 16347 18708 16355 18737  
18765 16983 18761 18715 18729 18714  
18756 16303 16356 18709 16357 18743  
18758 16987 16989 18744 16988 18745  
18757 18755 18760 18747 18734 18733  
18755 16527 18700 18710 18701 18749  
18755 18696 18760 18748 18738 18734  
18702 18764 18763 18720 18718 18722  
18761 16983 16985 18729 16984 18730  
18755 18698 18696 18750 18697 18748  
18763 18760 18692 18726 18731 18723  
16356 18759 18756 18708 18741 18743  
18761 16985 16987 18730 16986 18707  
18756 16991 16303 18752 16992 18709  
18756 18758 16989 18742 18745 18751  
16991 18756 16989 18752 18751 16990  
18702 155 18764 18704 18721 18720  
155 157 18764 156 18719 18721  
18755 16536 16527 18754 16535 18710  
18765 143 16983 18717 16982 18715  
18762 157 18765 18724 18716 18713  
18694 18760 18696 18739 18738 18695  
18755 18757 16347 18747 18746 18753  
16987 18758 18761 18744 18735 18707  
16347 16536 18755 16537 18754 18753  
18765 18761 18762 18714 18727 18713  
18763 18692 18690 18723 18691 18711  
18759 16347 18757 18737 18746 18732  
18763 18762 18760 18725 18705 18726  
157 18762 18764 18724 18712 18719  
18757 18760 18762 18733 18705 18728  
18762 18761 18757 18727 18706 18728  
18761 18759 18757 18736 18732 18706

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126 142 141 140 139 138 137 136 135 134  
133 6704 6705 6706 6707 6708 6709 6710 6711 6712  
6713 6714 6715 6716 6717 6718 6719 6720 6721 6722  
6723 6724 6725 6726 6727 6728 6729 6730 6731 6732  
6733 6734 6735 6736 6737 6738 6739 6740 6741 6742  
6743 6744 6745 6746 6747 6748 6749 6750 6751 6752  
6753 6754 6698

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5857 7564 7565 7566 6698

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35 14 5913 5912 5911 5910 5909 5908 5907 5906  
5905 5904 5903 5902 5901 5900 5899 5898 5897 5896  
5895 5894 5893 5892 5891 5890 5889 5888 5887 5886  
5885 5884 5883 5882 5881 5880 5879 5878 5877 5876  
5875 5874 5873 5872 5871 5870 5869 5868 5867 5866  
5865 5864 5863 5857

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5857 5858 5859 5860 5861 5862 5845 5856 5855 5854  
5853 5852 5851 5850 5849 5848 5847 5846 5801 8862  
8863 8864 8865 8866 8867 8868 8869 8870 8871 8872  
8861 8877 8876 8875 8874 8873 2015

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6698 6699 6700 6701 6702 6703 6686 6697 6696 6695  
6694 6693 6692 6691 6690 6689 6688 6687 6642 9616  
9615 9614 9613 9612 9611 9610 9609 9608 9607 9606  
9605 9617 9618 9619 9620 9621 2067

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2015 2068 2069 2070 2067

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2015 2016 2017 2018 2019 2020 2021 2022 2023 2024  
2025 2026 2027 2028 2029 2030 2031 2032 2033 2034  
2035 2036 2037 2038 2039 2040 2041 2042 2043 2044  
2045 2046 2047 2048 2049 2050 2051 2052 2053 2054  
2055 2056 2057 2058 2059 2060 2061 2062 2063 2064  
2065 2066 2005 2014 2013 2012 2011 2010 2009 2008  
2007 2006 2001

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2067 2121 2120 2119 2118 2117 2116 2115 2114  
2113 2112 2111 2110 2109 2108 2107 2106 2105 2104  
2103 2102 2101 2100 2099 2098 2097 2096 2095 2094  
2093 2092 2091 2090 2089 2088 2087 2086 2085 2084  
2083 2082 2081 2080 2079 2078 2077 2076 2075 2074  
2073 2072 2071 1990 1992 1993 1994 1995 1996 1997  
1998 1999 2000 1991

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2001 2002 2003 2004 1991

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2001 5310 5311 5312 5313 5314 5302 5309 5308 5307  
5306 5305 5304 5303 5298 17961 17960 17974 17962

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1991 5428 5427 5426 5425 5424 5416 5423 5422 5421  
5420 5419 5418 5417 5412 18039 18036 18038 18037

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17962 18126 18127 18128 18037

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17962 17963 17964 17965 17966 17967 17968 17969 17970  
17971 17972 17973 16559 16613 16612 16611 16595

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16647 16705 16704 16703 16702 16701 16700 16699 16595

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18037 18050 18049 18048 18047 18046 18045 18044 18043 18042  
18041 18040 16563 16648 16649 16650 16647

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18690 18691 18692 18693 18694 18695 18696 18697 18698 18699  
18700 18701 16527 16663 16664 16665 16647

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18616 18617 18618 18619 18620 18621 18622 18623 18624 18625  
18626 18627 16523 16598 16597 16596 16595

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18616 18780 18781 18782 18690

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18616 18628 18614 18615 2 9 8 7 6 5  
4 3 1 40 39 38 37 36 26

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18690 18703 18702 18704 155 165 164 163 162 161  
160 159 127 132 131 130 129 128 126

## CRACK3D.SIF

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.00000E+00 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.63500E-01	0.00000E+00	-0.23812E-02	0.37165E-03	0.10656E-03	0.37377E-05	0.54455E+04	-0.29159E+04	0.44700E+03
2	-0.63500E-01	0.00000E+00	-0.79375E-03	0.39629E-03	0.10753E-03	0.46060E-06	0.56231E+04	-0.29291E+04	0.15692E+03
3	-0.63500E-01	0.00000E+00	0.79375E-03	0.37837E-03	0.10506E-03	0.34734E-06	0.54945E+04	-0.28952E+04	-0.13627E+03
4	-0.63500E-01	0.00000E+00	0.23812E-02	0.38625E-03	0.11748E-03	0.46666E-05	0.55514E+04	-0.30616E+04	-0.49947E+03

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.20000E-02 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.62004E-01	0.13276E-02	-0.23812E-02	0.75121E-03	0.31880E-06	0.16848E-06	0.77420E+04	-0.15949E+03	0.94904E+02
2	-0.62004E-01	0.13276E-02	-0.79375E-03	0.79535E-03	0.36949E-06	0.31913E-07	0.79662E+04	-0.17170E+03	0.41304E+02
3	-0.62004E-01	0.13276E-02	0.79375E-03	0.79535E-03	0.34982E-06	0.32934E-07	0.79662E+04	-0.16707E+03	-0.41959E+02
4	-0.62004E-01	0.13276E-02	0.23813E-02	0.77147E-03	0.29758E-06	0.19412E-06	0.78457E+04	-0.15409E+03	-0.10187E+03

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.40000E-02 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.60510E-01	0.26575E-02	-0.23812E-02	0.89194E-03	0.62888E-06	0.12236E-08	0.84361E+04	-0.22400E+03	0.80879E+01
2	-0.60510E-01	0.26575E-02	-0.79375E-03	0.94345E-03	0.68460E-06	0.35363E-08	0.86762E+04	-0.23372E+03	0.13749E+02
3	-0.60510E-01	0.26575E-02	0.79375E-03	0.94319E-03	0.84510E-06	0.33224E-08	0.86751E+04	-0.25967E+03	-0.13327E+02

4 -0.60510E-01 0.26575E-02 0.23813E-02 0.91714E-03 0.75434E-06 0.40909E-08 0.85544E+04  
0.24533E+03 -0.14788E+02

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.60000E-02 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.58945E-01	0.39018E-02	-0.23812E-02	0.10424E-02	0.24602E-08	0.23384E-07	0.91198E+04	-0.14011E+02	0.35356E+02
2	-0.58945E-01	0.39018E-02	-0.79375E-03	0.11053E-02	0.63708E-08	0.30365E-08	0.93908E+04	-0.22546E+02	0.12741E+02
3	-0.58945E-01	0.39018E-02	0.79375E-03	0.11061E-02	0.71879E-08	0.57638E-08	0.93942E+04	-0.23948E+02	-0.17554E+02
4	-0.58945E-01	0.39018E-02	0.23813E-02	0.10718E-02	0.37370E-07	0.32012E-07	0.92474E+04	-0.54605E+02	-0.41368E+02

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.80000E-02 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.57387E-01	0.51558E-02	-0.23812E-02	0.11912E-02	0.32908E-08	0.11088E-07	0.97489E+04	-0.16204E+02	0.24346E+02
2	-0.57387E-01	0.51558E-02	-0.79375E-03	0.12643E-02	0.70049E-08	0.29710E-08	0.10044E+05	-0.23641E+02	0.12603E+02
3	-0.57387E-01	0.51558E-02	0.79375E-03	0.12599E-02	0.12317E-07	0.38677E-08	0.10026E+05	-0.31349E+02	-0.14379E+02
4	-0.57387E-01	0.51558E-02	0.23813E-02	0.12210E-02	0.15173E-07	0.23911E-07	0.98704E+04	-0.34794E+02	-0.35752E+02

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.10000E-01 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.55835E-01	0.64181E-02	-0.23812E-02	0.13308E-02	0.10571E-07	0.87559E-08	0.10305E+05	-0.29042E+02	0.21635E+02
2	-0.55835E-01	0.64181E-02	-0.79375E-03	0.14127E-02	0.86960E-08	0.20416E-09	0.10617E+05	-0.26341E+02	0.33036E+01

3 -0.55835E-01 0.64181E-02 0.79375E-03 0.14136E-02 0.24837E-08 0.10044E-08 0.10620E+05 -  
0.14077E+02 -0.73275E+01  
4 -0.55835E-01 0.64181E-02 0.23813E-02 0.13692E-02 0.14156E-07 0.10934E-07 0.10452E+05 -  
0.33608E+02 -0.24177E+02

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.12000E-01 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.54287E-01	0.76841E-02	-0.23812E-02	0.14626E-02	0.92988E-08	0.20640E-08	0.10803E+05	0.27239E+02	0.10504E+02
2	-0.54287E-01	0.76841E-02	-0.79375E-03	0.15574E-02	0.18122E-07	0.93540E-09	0.11147E+05	0.38026E+02	0.70714E+01
3	-0.54287E-01	0.76841E-02	0.79375E-03	0.15557E-02	0.26493E-07	0.37783E-09	0.11141E+05	0.45977E+02	-0.44943E+01
4	-0.54287E-01	0.76841E-02	0.23813E-02	0.15045E-02	0.11177E-08	0.29300E-08	0.10956E+05	0.94435E+01	-0.12515E+02

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.14000E-01 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.52732E-01	0.89416E-02	-0.23812E-02	0.15860E-02	0.16973E-07	0.14073E-08	0.11249E+05	0.36800E+02	0.86736E+01
2	-0.52732E-01	0.89416E-02	-0.79375E-03	0.16916E-02	0.19714E-07	0.59647E-09	0.11618E+05	0.39661E+02	0.56468E+01
3	-0.52732E-01	0.89416E-02	0.79375E-03	0.16832E-02	0.10127E-07	0.18859E-08	0.11589E+05	0.28426E+02	-0.10041E+02
4	-0.52732E-01	0.89416E-02	0.23813E-02	0.16344E-02	0.22974E-07	0.12336E-08	0.11420E+05	0.42814E+02	-0.81208E+01

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.16000E-01 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.51168E-01	0.10189E-01	-0.23812E-02	0.17012E-02	0.39117E-07	0.71751E-09	0.11651E+05	0.55867E+02	0.61933E+01

2 -0.51168E-01 0.10189E-01 -0.79375E-03 0.18082E-02 0.36286E-07 0.63876E-10 0.12011E+05  
0.53807E+02 0.18479E+01  
3 -0.51168E-01 0.10189E-01 0.79375E-03 0.18068E-02 0.82220E-07 0.81703E-09 0.12007E+05  
0.80995E+02 -0.66089E+01  
4 -0.51168E-01 0.10189E-01 0.23813E-02 0.17491E-02 0.21554E-07 0.11258E-08 0.11813E+05  
0.41470E+02 -0.77578E+01

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.18000E-01 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.49593E-01	0.11421E-01	-0.23812E-02	0.18029E-02	0.20153E-07	0.11729E-08	0.11994E+05	0.40100E+02	0.79183E+01
2	-0.49593E-01	0.11421E-01	-0.79375E-03	0.19172E-02	0.34006E-07	0.71986E-09	0.12368E+05	0.52089E+02	-0.62034E+01
3	-0.49593E-01	0.11421E-01	0.79375E-03	0.19172E-02	0.55772E-07	0.20859E-08	0.12368E+05	0.66708E+02	0.10560E+02
4	-0.49593E-01	0.11421E-01	0.23813E-02	0.18547E-02	0.26017E-07	0.19943E-09	0.12165E+05	0.45561E+02	-0.32652E+01

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.20000E-01 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.48007E-01	0.12640E-01	-0.23812E-02	0.18917E-02	0.66330E-07	0.27124E-09	0.12286E+05	0.72749E+02	-0.38079E+01
2	-0.48007E-01	0.12640E-01	-0.79375E-03	0.20160E-02	0.71559E-07	0.14384E-09	0.12683E+05	0.75562E+02	-0.27730E+01
3	-0.48007E-01	0.12640E-01	0.79375E-03	0.20118E-02	0.88642E-07	0.24471E-08	0.12670E+05	0.84099E+02	0.11438E+02
4	-0.48007E-01	0.12640E-01	0.23813E-02	0.19478E-02	0.50861E-07	0.80686E-09	0.12466E+05	0.63703E+02	0.65676E+01

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.22000E-01 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
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1 -0.46407E-01 0.13840E-01 -0.23812E-02 0.19787E-02 0.69089E-07 0.29344E-09 0.12565E+05  
0.74246E+02 -0.39607E+01  
2 -0.46407E-01 0.13840E-01 -0.79375E-03 0.21077E-02 0.85531E-07 0.31981E-09 0.12968E+05  
0.82610E+02 -0.41348E+01  
3 -0.46407E-01 0.13840E-01 0.79375E-03 0.21035E-02 0.13675E-06 0.24518E-08 0.12955E+05  
0.10446E+03 0.11449E+02  
4 -0.46407E-01 0.13840E-01 0.23813E-02 0.20370E-02 0.87836E-07 0.13225E-10 0.12749E+05  
0.83716E+02 0.84083E+00

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.24000E-01 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.44791E-01	0.15019E-01	-0.23812E-02	0.20479E-02	0.12539E-06	0.50449E-08	0.12783E+05	0.10002E+03	-0.16422E+02
2	-0.44791E-01	0.15019E-01	-0.79375E-03	0.21819E-02	0.18469E-06	0.57208E-09	0.13194E+05	0.12139E+03	0.55301E+01
3	-0.44791E-01	0.15019E-01	0.79375E-03	0.21824E-02	0.19846E-06	0.24257E-08	0.13196E+05	0.12584E+03	0.11387E+02
4	-0.44791E-01	0.15019E-01	0.23813E-02	0.21062E-02	0.15600E-06	0.52980E-09	0.12964E+05	0.11157E+03	0.53219E+01

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.26000E-01 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.43155E-01	0.16168E-01	-0.23812E-02	0.21119E-02	0.88676E-07	0.30555E-08	0.12981E+05	0.84115E+02	-0.12781E+02
2	-0.43155E-01	0.16168E-01	-0.79375E-03	0.22547E-02	0.10418E-06	0.74047E-10	0.13413E+05	0.91173E+02	-0.19896E+01
3	-0.43155E-01	0.16168E-01	0.79375E-03	0.22510E-02	0.71895E-07	0.49060E-09	0.13402E+05	0.75739E+02	-0.51212E+01
4	-0.43155E-01	0.16168E-01	0.23813E-02	0.21712E-02	0.68106E-07	0.12286E-08	0.13162E+05	0.73717E+02	0.81041E+01

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.28000E-01 Multiplicative factor used to scale loading: 0.10000E+01



SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.41505E-01	0.17298E-01	-0.23812E-02	0.21698E-02	0.20511E-06	0.17686E-08	0.13158E+05	0.12793E+03	-0.97236E+01
2	-0.41505E-01	0.17298E-01	-0.79375E-03	0.23086E-02	0.25158E-06	0.96179E-09	0.13572E+05	0.14168E+03	-0.71705E+01
3	-0.41505E-01	0.17298E-01	0.79375E-03	0.23055E-02	0.34365E-06	0.18926E-08	0.13563E+05	0.16559E+03	0.10059E+02
4	-0.41505E-01	0.17298E-01	0.23813E-02	0.22313E-02	0.23404E-06	0.65428E-08	0.13343E+05	0.13665E+03	0.18702E+02

LOADING INCREMENT NUMBER = 1

Energy release rates & SIFs for crack front#: 1  
(This is an open crack front)

Total crack extension so far: 0.30000E-01 Multiplicative factor used to scale loading: 0.10000E+01

SEGMT#	X	Y	Z	G1	G2	G3	K1	K2	K3
1	-0.39831E-01	0.18392E-01	-0.23812E-02	0.22169E-02	0.12940E-06	0.46224E-08	0.13300E+05	0.10161E+03	-0.15720E+02
2	-0.39831E-01	0.18392E-01	-0.79375E-03	0.23662E-02	0.18496E-06	0.40042E-09	0.13740E+05	0.12148E+03	-0.46266E+01
3	-0.39831E-01	0.18392E-01	0.79375E-03	0.23638E-02	0.18353E-06	0.38330E-08	0.13733E+05	0.12101E+03	0.14315E+02
4	-0.39831E-01	0.18392E-01	0.23813E-02	0.22830E-02	0.20926E-06	0.15014E-09	0.13497E+05	0.12922E+03	0.28331E+01

## CRACK3D.CXT

Arcan 4 elem through thickness

### INFORMATION OF LOAD VERSUS TOTAL CRACK INCREMENT

AT SPECIFIED MASTER NODE POSITION

( Reference Z-COORDINATE: 0.00000 )

LOADING		CRACK FRONT	TOTAL CRACK	TOTAL LOADS ON ALL SPECIFIED NODES			
INCREMENT	NUMBER	INCREMENT	X-LOAD	Y-LOAD	Z-LOAD	COMBINED	
1	1	0.00000	0.21361E+02	-0.21362E+02	-0.29745E-12	0.30210E+02	
1	1	0.00200	0.21236E+02	-0.21236E+02	-0.42413E-08	0.30033E+02	
1	1	0.00400	0.21086E+02	-0.21086E+02	-0.39401E-08	0.29820E+02	
1	1	0.00600	0.20909E+02	-0.20909E+02	0.35147E-08	0.29570E+02	
1	1	0.00800	0.20704E+02	-0.20705E+02	-0.41428E-09	0.29281E+02	
1	1	0.01000	0.20473E+02	-0.20473E+02	-0.12057E-08	0.28953E+02	
1	1	0.01200	0.20216E+02	-0.20216E+02	-0.35862E-08	0.28590E+02	
1	1	0.01400	0.19935E+02	-0.19936E+02	0.14178E-07	0.28193E+02	
1	1	0.01600	0.19633E+02	-0.19633E+02	0.53847E-09	0.27765E+02	
1	1	0.01800	0.19310E+02	-0.19311E+02	0.18068E-08	0.27309E+02	
1	1	0.02000	0.18970E+02	-0.18971E+02	0.28869E-08	0.26828E+02	
1	1	0.02200	0.18614E+02	-0.18614E+02	0.45479E-08	0.26324E+02	
1	1	0.02400	0.18243E+02	-0.18244E+02	0.32076E-08	0.25800E+02	
1	1	0.02600	0.17860E+02	-0.17861E+02	0.27318E-08	0.25259E+02	
1	1	0.02800	0.17466E+02	-0.17467E+02	0.89362E-09	0.24701E+02	
1	1	0.03000	0.17062E+02	-0.17063E+02	-0.32270E-08	0.24130E+02	